

Return period analysis of extreme rainfall events

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ABSTRACT. The return periods analysis of extreme events of rainfall for selected 15 stations of Krishna river basin have been studied and presented in this paper. The annual one-day rainfall data for 60-years (1901-1960) was used for the study. The rainfall estimates were computed for different return periods using two different distributions, namely, Gumbel’s distribution and Fisher & Tipett type II distribution. Return periods corresponding to highest recorded rainfall were interpolated for both the distributions. Results show that while Gumbel’s distribution seems to indicate very high return periods of extreme events, the Fisher & Tipett type-II distribution provides lower estimates of the recurrence interval. As rainfall events are randomly distributed in nature, the return periods of outliers should also be determined with reasonable accuracy. The present study shows that the rainfall estimates for different return periods as obtained by using Fisher & Tipett type II distribution technique are much higher than those obtained by using Gumbel’s technique. It is suggested that Fisher & Tipett type II distribution may be preferred for evaluating the return periods of extreme rainfall events to Gumbel distribution.

I. Introduction

In the design of water-way engineering structures the twin factors of safety and economy are both important as in other engineering structures. By safety of the structures, it is implied that the structure should be able to withstand the rainstorms in the catchment or over the site itself in a given period. While safety takes precedence over all other considerations, it is an indication of the technological progress to design a structure in an optimum cost benefit ratio. The return period estimates of the maximum rainfall likely to occur are used by the hydraulic engineers for the design purposes. The optimum design of a structure is made by striking a balance between a calculated risk on estimated return periods of rainfall and the constraints on the availability of funds. The maximum rainfall estimates for specified periods can be made by using suitable extreme value distributions (India Met. Dep. 1972).

Rao and Krishnan (1958) applied Gumbel’s and Jenkinson’s methods for the determination of maximum probable rainstorms. They applied the above methods of the data of Damodar river catchment for the period 1891 to 1950. They computed rainfall estimates for various return periods for the rainstorms of 5 to 7 days. The comparison of the results by two methods showed marked difference between the computed and observed frequencies. They suggested that all rainfall amounts rather than extreme values should be considered while evaluating probabilities. Harivaraha Ayyar and Tripathi (1971) studied the heaviest rainfall over India recorded in relation to return period. For the purpose, 50 stations all over the country and 50 years one day annual maximum rainfall data were utilised and applying Gumbel’s distribution to the series the return period values for the extreme events were computed by them. The return periods of the extreme events presented by above authors vary from 21 to 2883 years; they did not discuss the implication of this wide range, and left it to the users to decide appropriate methodology for selecting the design value. This gives rise to an under-estimation of the rainfall in a given return period for design criterion endangering the
Fig. 1. Locations of stations used in the study

Fig. 2. Graph showing estimated rainfall in respect of 3 selected stations for different return periods
safety of structures. The reason for low value is that
Gumbel’s extreme value distribution does not estimate
outliers, within reasonable degree of accuracy.

Dhar and Kulkarni (1970) estimated one-day rain-
fall estimates for different return periods in Uttar
Pradesh. They selected 226 stations in plains of Uttar
Pradesh and computed the 2-year and 100-year return
periods rainfall estimates. They presented generalised
chart of 2-year one-day rainfall. The ratios of 100 to
2-year rainfall estimates were worked out for the dis-
tricts in UP. However, the problem of outliers was not
discussed.

In the present study, an attempt has, therefore, been
made to estimate the rainfall values for different return
periods, by two extreme value distributions, namely
Gumbel’s distribution and Fisher & Tippett Type-II
distribution. The annual rainfall series were tested for
the outliers. If the extreme value exceeded 3 times
the median value of the data set, that value was
replaced as off-shoot or an outlier (UK, Flood Report
1975).

2. Data used

The input variate for the study has been taken as
the one day annual maximum rainfall. Fifteen stations
in Krishna river basin (Fig. 1) were considered and
60 years (1901-60) rainfall data have been utilised.

3. Return period analysis

The probability density functions used for the esti-
mation of return periods corresponding to the extreme
rainfall recorded over a station, are given below :

(a) Using Gumbel distribution

Let \( x_1, x_2, \ldots, x_n \) be the extreme value
annual series of one day rainfall over the station.
Gumbel’s distribution (1958) is given by :

\[
F(x) = e^{e^{-\frac{x-u}{\alpha}}}
\]

(1)

where the parameters \( \alpha \) and \( u \) may be estimated using the
method of moments as suggested by WMO (1981)
leading to relation :

\[
\alpha = 1/0.78 \bar{x}
\]

(2)

and

\[
u = \bar{x} - 0.58/\alpha
\]

(3)

Here \( \bar{x} \) and \( s \) are the mean and standard deviation
of the extreme rainfall series.

The rainfall estimate \( x_T \) for a given return period
\( T \) may be computed by :

\[
x_T = u + y_T/\alpha
\]

(4)

where \( y_T \) is the reduced variate given by

\[
y_T = -\log_e \log_e \left( \frac{T}{T - 1} \right)
\]

(5)

(b) Fisher and Tippett type II distribution

The type II distribution is also known as log Gumbel
distribution (Flood Report 1975). The density function
of \( x \) is given by

\[
F(x) = \exp \left[ -e^{-\alpha (z - u)} \right]
\]

(6)

and where \( z = \log x \). The parameters \( \alpha \) and \( u \) are
estimated by method of moments. These are :

\[
\alpha = 1/0.78 \bar{s}
\]

(7)

and

\[
u = z - 0.58/\alpha
\]

(8)

where \( \bar{z} \) & \( s \) are the mean & standard deviation of the
log of rainfall series.

The rainfall estimate \( x_T \) for any given return period
\( T \) may be computed by :

\[
x_T = \text{antilog} \left( \bar{u} + y_T/\alpha \right)
\]

(9)

where \( y_T \) is given by Eqn. (5). The rainfall estimates
as computed from Eqn. (9) for various return periods
are given in Table 1. The return periods of highest
recorded rainfall as estimated by this technique are
shown in Table 2.

4. Standard errors of quantiles estimates

From Eqn. (4) we find that \( x_T \) is a function of mean
\( \bar{x} \) and standard deviation \( s \) :

\[
x_T = x_T (\bar{x}, s)
\]

It is asymptotically normally distributed with variance
(Flood study Rep. 1975) given by :

\[
\begin{align*}
\text{var } x_T &= \frac{\partial x_T}{\partial \bar{x}}^2 \text{var } \bar{x} + \frac{\partial x_T}{\partial s^2}^2 \text{var } s^2 + 2 \frac{\partial x_T}{\partial \bar{x}} \frac{\partial x_T}{\partial s} \text{cov } (\bar{x}, s^2) \\
\frac{\partial x_T}{\partial \bar{x}} &= 1, \quad \frac{\partial x_T}{\partial s^2} = \frac{-0.45 + 0.78 y_T \alpha}{2}
\end{align*}
\]

where the partial derivatives are evaluated at the expected
values of \( \bar{x} \) and \( s^2 \) and are given by :

\[
\begin{align*}
\text{var } \bar{x} &= \frac{1}{0.78^2 s^2 N} \\
\text{var } s^2 &= \frac{4.40}{(0.78 s)^4 N} \\
\text{cov } (\bar{x}, s^2) &= \frac{1.14}{(0.78 s)^3 N}
\end{align*}
\]

The variance and covariance terms are given by :

\[
\begin{align*}
\text{var } \bar{x} &= \frac{1}{0.78^2 s^2 N} \\
\text{var } s^2 &= \frac{4.40}{(0.78 s)^4 N} \\
\text{cov } (\bar{x}, s^2) &= \frac{1.14}{(0.78 s)^3 N}
\end{align*}
\]

The moment ratios \( \mu_3/\mu_2^{3/2} \) & \( \mu_4/\mu_2^2 \) are the co-
efficients of skewness & kurtosis which are 1.14
### TABLE 1
Return period estimates of rainfall (mm)

<table>
<thead>
<tr>
<th>S. No.</th>
<th>Station</th>
<th>$x$-25yr</th>
<th>$x$-50yr</th>
<th>$x$-100yr</th>
<th>$x$-150yr</th>
<th>$x$-200yr</th>
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G — Gumbel's estimate  
T - II — Fisher & Tipett type II  
x — Rainfall estimates

### TABLE 2
Return period values of the extreme events

<table>
<thead>
<tr>
<th>S. No.</th>
<th>Station</th>
<th>Highest rainfall (mm)</th>
<th>Date</th>
<th>Gumbel's method parameters</th>
<th>Type-II parameters</th>
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<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Mean $x$ (mm)</td>
<td>St. Dev. $\sigma$ (mm)</td>
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<td>1</td>
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<td>12-6-1991</td>
<td>112.03</td>
<td>44.36</td>
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<td>14-9-1959</td>
<td>112.46</td>
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<td>1-8-1954</td>
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<td>10-7-1975</td>
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<td>42.23</td>
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<td>Poona</td>
<td>149.1</td>
<td>14-9-1982</td>
<td>69.89</td>
<td>23.80</td>
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<td>21-5-1940</td>
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<td>29-9-1908</td>
<td>91.84</td>
<td>47.44</td>
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<td>18-9-1970</td>
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<td>191.0</td>
<td>12-8-1940</td>
<td>80.19</td>
<td>29.66</td>
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TABLE 3

Standard error (mm) of the rainfall estimates for different return periods (yr)

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<tr>
<th>S. No.</th>
<th>Station</th>
<th>S.E. G</th>
<th>25 yrs T-II</th>
<th>S.E. G</th>
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<th>S.E. G</th>
<th>150 yrs T-II</th>
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<td>61.2</td>
<td>24.3</td>
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TABLE 4

Confidence bands of return periods of extreme events

<table>
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<th>S. No.</th>
<th>Station</th>
<th>Highest rainfall on record up to 1979 (mm)</th>
<th>Median (mm)</th>
<th>3x median (mm)</th>
<th>Gumbel upper limit (ys)</th>
<th>Type II upper limit (ys)</th>
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*For outliers

and 4.40. Using above values, \( x_T \) can be computed and the standard error of estimate is given by:

\[
\text{S.E.} (x_T) = \frac{1}{a} \left( 1.17 + 0.196 y_T + 1.099 y_T^2 \right)^{1/2} / N
\]

where \( y_T \) is reduced variate given by Eqn. (5).

If \( x_T \) is estimated from type II distribution, then the standard error estimate can be computed by:

\[
\text{S.E.} (x_T) = 1 \left( 1.17 + 0.196 y_T + 1.099 y_T^2 \right)^{1/2} / N
\]

where \( y_T \) is the rainfall estimate.

5. Results and discussion

5.1. Gumbel distribution

The 60 years (1901-60) data of 15 stations of Krishna river basin were analysed by Gumbel and Fisher & Tipett type II distributions. Rainfall estimates for return periods of 25, 50, 100, 150, 200 and 500-year were computed and listed in Table 1 (lower return periods not shown). Corresponding return periods of extreme events on record were interpolated by using both methods, which may be seen in Table 2. According to Gumbel's estimate, return periods of extreme event on record varies from 88 years to 4830 years. Such a large variation in return periods of extreme events on record suggest need for trying an alternative methodology.

As a matter of fact, the Gumbel's distribution does not give correct rainfall estimates for various return periods for the rainfall series containing outliers. The outliers were worked out as a value exceeding 3 times the median of the data series. These outliers are listed in Table 4. It may be mentioned that 5 out of 15 stations fell in the category of outliers. The Gumbel's estimate of return periods of these outliers varies from 8 to 80 times of the length of the series. This shows that these outliers should be treated with an alternative method. The authors suggest the application of Fisher Tipett type II distribution for the computation of rainfall.
estimates for various return periods for those stations having outliers in the series in order to avoid under estimation of design rainfall which may otherwise have serious consequences.

5.2. Fisher & Tippett Type II distribution

Application of Fisher & Tippett type II distribution for estimation of return periods of extreme events was made (Table 2). The return period estimates for outliers varies from 70 to 164 years.

It may be added that the above distributions can be expressed by the equation:

\[ x = a (1 - e^{-ky}) \]

where \( k = 0 \), represents Gumbel's distribution and \( k < 0 \) stands for type II distribution. Gumbel's distribution when plotted is a straight line fit and type II is a curve with no upper bound. Perhaps that is why the return period estimates are very high in case of type II distribution beyond return periods of 50 years.

5.3. Standard error of estimates

The rainfall events are random in nature and at 67% level of confidence are expected to be within the limits of variate \( \pm \) standard error. Therefore, standard errors were computed for extremes under the probability density functions given by Gumbel and type II for all the return periods. These are shown in Table 3. By using both methods, return periods were also computed for the rainfall events (extreme on record) for lower and higher limits of extreme rainfall, i.e., variate \( \pm \) S.E. These are shown in Table 4. The lower limit of the variates are already covered under the extreme events itself and its return period carries no significance for statistical analysis of extremes. The return period of upper limit have further gone up in both methods. The type II values vary from 118 years to 329 years for outliers and also for significant extreme.

The fitting of Gumbel and type II distributions have been shown in Fig. 2. Three stations were chosen taking into account highest event, lowest amongst outliers and a medium event from series. Graphical representation is shown up to a return period of 100-yr only. From the graphs it can be seen that for lower values of return periods, say less than 25 years, the estimate by both methods do not differ significantly. This suggests that wherever return period values of 25 years or less are required by design engineers, Gumbel's method can be applied safely. Further, the graphs showed a large and significant variation for the higher return periods of 50-year or 100-year which are most needed for design purposes. For such return periods due cognisance should be made of the events on record and if the highest or 2nd highest event of a series corresponds to a very high return period (more than 4 to 5 times the length of the series) Gumbel's technique should not be applied. Else, the series may be treated by type II distribution for high return period estimates of 50-year or 100-year.

6. Conclusions

The following conclusions can be made from the study:

1. For return periods of more than 25 years Gumbel's technique does not estimate some highest rainfall events within acceptable degree of accuracy, for certain conditions of rainfall occurrence.

2. If Gumbel's method shows a return period of an extreme events on record more than 4 to 5 times of the length of series, type II distribution may be applied to estimate design rainfall for required return periods (50 years or more).

3. Test for outliers should be made before application of any method for return period analysis.

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References


