Vertical potential gradient profile under point discharge current and wind

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ABSTRACT. An attempt is made to study the influence of winds and point discharge current on vertical electric potential gradient profile in thunderstorms in the atmosphere. It is further extended to the case of precipitation where falling drops possess charge but do not interact with ions. The profiles under different wind speeds are plotted and compared.

1. Introduction

Chalmers (1944) studied the vertical potential gradient profile in determination of the electric charges carried down by the raindrops in a thunderstorm. The potential gradient was found to increase with height but Chalmers neglected the space charge on the rain being small when compared with the space charge originated from point discharge current. Lane-Smith (1972) attempted this problem in a more analytical way, but he neglected the effect of large scale vertical wind velocities on potential gradient profile.

The present attempt includes the effect of wind velocities on vertical profile of potential gradient. The work is presented in two stages. The first case deals with point discharge currents with winds but no precipitation. In the second case precipitation is included with the assumption that while the drops fall, they do not collect ions.

2. Analysis

Case I

Under conditions of winds but no precipitation the variation of potential gradient with height can be calculated from the following equations, with a uniform point discharge current over a wide area,

\[ J = ne \sqrt{(\omega E)^2 + W^2} \]  
\[ \frac{\partial E}{\partial z} = -\frac{ne}{\epsilon_0} \]  

where \( J \) is point discharge current density,
\( n \) number of small ions of charge \( e \),
\( \omega \) mobility of small ions
\( E \) potential gradient, \( z \) height, \( \epsilon_0 \) permittivity of air and
\( W \) is the wind speed.

In the above equation we have not considered the contribution of large ions.

It is not improper to include the effect of winds in Eq. (1) as point discharge current is greatly influenced by winds. It has been shown by many workers (Chalmers 1944, Large and Pierce 1957, Verma and Varshneya 1974) that net velocity of the small ions is the vector sum of electrical velocity and wind velocity.

Assuming \( J \) and \( \omega \) independent of height, the solution of above equation becomes

\[ \frac{1}{2} \frac{\omega E^2}{\epsilon_0} \sqrt{1 + \frac{W^2}{\omega E^2}} + \frac{1}{2} \frac{W^2}{\omega E^2} \]

\[ \ln \left( E + E \sqrt{1 + \frac{W^2}{\omega^2 E^2}} \right) \]

\[ = -\frac{J}{\epsilon_0} z + \text{constant} \]  

(3)

The observations with tropical thunderstorm Lane-Smith (1972), Sivaramakrishnan (1961) show the existence of negative electric field of few KVm\(^{-1}\) or higher and point discharge current density \(-10^{-9} \text{ Arm}^{-2}\). Using some typical values,

\[ J = -2 \times 10^{-9} \text{ amp m}^{-2}, \]
\[ \omega = 2 \times 10^{-4} \text{ m}^2 \text{V}^{-1} \text{ sec}^{-1}, \]
\[ \epsilon_0 = 8.85 \times 10^{-12} \text{ Fm}^{-1}, \]
\[ E_0 = -3 \times 10^3 \text{ Vm}^{-1} \text{ and} \]

\[ W = 0, 1, 2, 4, 6 \ldots \text{m sec}^{-1} \], we get potential gradient profile as shown in Fig. 1.
Case II: Precipitation with no collection of charge

The Eqns. (1) and (2) are modified if one includes the charges brought down by the raindrops itself. It is assumed that drops collect no ions, hence Eqns. (1) and (2) become

\[ J = ne \sqrt{(\omega E)^2 + W^2} \tag{4} \]
\[ \frac{\partial E}{\partial z} = -\frac{1}{\varepsilon_0} (ne + NQ) \tag{5} \]

where \( N \) is number density of rain drops, \( J \) is conduction current density and \( Q \) is mean charge on a raindrop.

The solution of (4) and (5) gives

\[ \frac{J}{NQ\omega} = \ln \left[ E + E \sqrt{1 + \frac{W^2}{\omega^2 E^2}} \right] - \frac{J^2}{N^2Q^2\omega^2} \sqrt{\frac{J^2}{N^2Q^2\omega^2} - \frac{W^2}{\omega^2}} \]

\[ \ln \left[ \frac{J^2}{N^2Q^2\omega^2} - \frac{W^2}{\omega^2} \right] \]

\[ \frac{J^2}{NQ\omega} + E \sqrt{1 + \frac{W^2}{\omega^2 E^2}} \times \left\{ \frac{J^2/NQ\omega^2}{NQ\omega - E \sqrt{1 + \frac{W^2}{\omega^2 E^2}}} \right\}^{1/2} - E \]

\[ = \frac{NQ}{\varepsilon_0} z + \text{constant} \tag{6} \]

The precipitation current density at the ground can be written as

\[ i_0 = NQU \]

where \( U \) is the average fall velocity of the raindrop.

Lane-Smith (1969) and Sivaramakrishnan (1961) measured in a tropical thunderstorm \( i_0 \) to be \( 2 \times 10^{-9} \) amp m\(^{-2} \) and \( U \) to be 7 m sec\(^{-1} \) (Fletcher 1962). Using same values of the parameters as used for Eqn. (3) together with the value of \( NQ = 3 \times 10^{-10} \) Coul. m\(^{-3} \) one would obtain potential gradient profile as shown in Fig. 2.
3. Results

The potential gradient profiles plotted in Figs. 1 and 2 under different conditions of winds are compared. The curves for potential gradient profiles are found to tend to be straight as wind speed increases beyond 4 m sec\(^{-1}\), whereas on the other hand, the potential gradient is found to be enhanced when precipitation is included along with wind velocity and the effect is strong at lower velocities.

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