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A non-linear surface layer model

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सार -- मिश्रण लम्बाई और वायु की प्रति इकाई संहति वाली कुल (गतिज एवं स्थैतिज) ऊर्जा के परस्पर सम्बन्ध के आधार पर पृष्ठ परत रिजाइम का अन्वेषण करने के लिए एक निदर्श विकसित किया गया है। निदर्श की प्रमुख विशेषताओं में प्रक्षुब्ध अभिलक्षणों के अ-रैखिक प्रोफाइल भी सम्मिलित है। उसका उपगामी आचरण क्लासकीय प्रक्षुब्ध निदर्शों के साथ काफी हद तक समानता का संकेत देता है।

ABSTRACT. A model has been developed to investigate surface layer regime on the basis of a relation between mixing length and the total (kinetic and potential) energy per unit mass of air. Important features of the model include non-linear profiles of turbulent characteristics. The provided asymptotic behaviour indicates considerable consistency with the classical turbulent models.

1. Introduction

Recognition and specification of the scale on which the vertical components of air motion exert their mixing are essential part of any general description of the boundary layer. There are various definitions of scale, the earliest and simplest form is the classical Prandtl relation between mixing length and distance from the boundary in the form :

$$l = kz.$$

For neutral boundary layer as a whole Blackadar (1962) has proposed :

$$l = \frac{kz}{1 + \frac{kz}{\lambda}} .$$

Blackadar's form is simply an interpolation form satisfying a tendency to kz at small z and to constant value at large z, reflecting the plausible hypothesis that the scale of the mixing ultimately becomes independent of the presence of the boundary.

In the diabatic case, Monin and Obukhov (1954) predicted that the velocity (and temperature) profiles in fully turbulent flow over a flat, homogeneous surface may be written :

$$\frac{\partial u}{\partial z} = \frac{u_*}{kz} \phi_M (z/L) ,$$

$$\frac{\partial \theta}{\partial z} = \frac{\theta_*}{kz} \phi_H (z/L)$$

Here θ is the mean potential temperature, u_* is the friction velocity, θ_* is the temperature-scaling parameter defined by :

$$\theta_* = - \frac{H}{\rho c_p u_*} ,$$

H being the vertical sensible heat flux, ρ the air density and c_p the specific heat of air at constant pressure. The Monin-Obukhov stability length is then defined by :

$$L = - \frac{u_*^3 T \rho c_p}{k g H}$$

g being the acceleration due to gravity. The logarithmic form of the profile is retained then if, at sufficiently small $|z/L|$ say

$$|z/L| \leqslant 0.01, \ \phi_M(z/L) = \phi_H(z/L) = \text{constant},$$

for the appropriate height range, which feature is revealed in a number of observational studies with $k = 0.41 \pm 0.01$ and $\phi_M = \phi_H = 1$, (*see, e.g.*, Pruit *et al.* 1973; Hicks 1976).

The relation between mixing length and wind shear followed from dimensional analysis is defined by Karman relation which may be written as :

$$l = \frac{-2k\left(\frac{du}{dz}\right)^2}{\frac{d}{dz}\left(\frac{du}{dz}\right)^2} \quad (1)$$

This implies that in the absence of free convection $\left(\frac{d\theta}{dz}=0\right)$, l can be a function of

$$\frac{du}{dz},\ \frac{d^2u}{dz^2}.$$

In a flow with buoyancy, Laikhtman and Zeletinkevich (1965) added g/T and the potential temperature gradient to the governing parameters of mixing length. On this base, a similar to Karman's relation have been proposed in the following form :

$$l=-\frac{2kC^{1/4}\psi}{\frac{d\psi}{dz}} \quad (2)$$

where,

$$\psi=\left(\frac{\partial u}{\partial z}\right)^2-\frac{g}{T}\frac{\partial\theta}{\partial z}.$$

Pasquill (1972) concludes from his discussion of scales that probably the most that can safely be stated is :

(*a*) In neutral flow λ_m/z is between 2 and 4 and effectively constant with height in the first 20 m or so [λ_m is the spectral scale].

(*b*) In neutral flow l_E/z is uncertain in the range $\frac{1}{3}$ to 2 over this height range [l_E is the Eulerian scale length].

(*c*) The effect of thermal stratification is to increase or decrease the scales in unstable or stable conditions, and in effect to increase or decrease the height range with effectively linear increase. However, the precise magnitudes of the effects on the scale and the existence and heights of maxima in the scales are still quite uncertain.

2. A non-linear model

In this section, a general equation relating the mixing length, l, and the total turbulent kinetic and potential energy per unit mass of air, E, is proposed. It is assumed that l is proportional to E and dE/dz which implies that :

$$l=\frac{-K\left[K\left(\frac{du}{dz}\right)^2-\frac{g}{T}K\frac{d\theta}{dz}\right]}{\frac{d}{dz}\left[K\left(\frac{du}{dz}\right)^2-\frac{g}{T}K\frac{d\theta}{dz}\right]}$$

This mixing length closure hypothesis includes the coefficient of turbulent exchange, K, and relates the mixing length to a physically significant property which is the total energy of turbulence. Also the proposed hypothesis avoids the uncertainty arised from the multipication of the term :

$$\frac{g}{T}\frac{\partial\theta}{\partial z}$$

by a factor of 2 in Zilitinkevich and Laikhtman hypothesis.

If we express the mean property $X(z)$ in the fully turbulent, constant flux layer as a scaling parameter X_0 and a dimensionless function X_n the complete system of equations may be written as follows :

$$\frac{K_0u_0}{L}K_n\frac{du_n}{dz_n}=u_*^2 \quad (4)$$

$$\frac{K_0\theta_0}{L}K_n\frac{d\theta_n}{dz_n}=-\frac{H}{\rho c_p} \quad (5)$$

$$\frac{K_0u_0^2}{L^2}K_n\left(\frac{du_n}{dz_n}\right)^2-\frac{gK_0\theta_0}{TL}K_n\frac{d\theta_n}{dz_n}-$$

$$-\frac{C\epsilon_0^{3/2}}{l_0}\frac{\epsilon_n^{3/2}}{l_n}=0 \quad (6)$$

$$K_0K_n=l_0\,\epsilon_0^{1/2}\,l_n\,\epsilon_n^{1/2} \quad (7)$$

$$l_0l_n=-\frac{kLE_n}{\frac{dE_n}{dz_n}} \quad (8)$$

where,

$$E=K\left(\frac{du}{dz}\right)^2-\frac{g}{T}K\frac{d\theta}{dz}$$

Eqn. (6) is the equation of local balance of turbulent energy per unit mass of air, ϵ being the rate of dissipation of turbulent kinetic energy per unit mass of air. Klebanoff (1954) determined the value of the constant $C=0.046$.

Evaluation of the scaling parameters is provided from the equalities :

$$\frac{K_0u_0}{L}=u_*^2,$$

$$\frac{K_0\theta_0}{L}=-\frac{H}{\rho c_p}$$

$$\frac{K_0u_0^2}{L^2}=\frac{g}{T}\frac{K_0\theta_0}{L}$$

$$=\frac{C\epsilon_0^{3/2}}{l_0}$$

and $K_0=l_0\,\epsilon_0^{\frac{1}{2}}$

The values of these scaling parameters will be equal to the values obtained from Monin-Obukhov similarity analysis if l_0 of Eqn. (8) is equal to $kLC^{1/4}$. This provides the following form for the proposed equation of mixing length :

$$l_0\, l_n = -\frac{kC^{1/4}\, L\, E_n}{\frac{dE_n}{dz_n}} \tag{9}$$

Replacing

$$\frac{du_n}{dz_n},\ \frac{d\theta_n}{dz_n} \text{ and } \epsilon_n$$

in the dimensionless form of Eqn. (6) with their values in the dimensionless forms of Eqns. (4), (5) and (7) and solving the resultant equation for l_n we get :

$$l_n = \frac{1 - X^4}{X} \tag{10}$$

where,

$$X = (1 - K_n)^{1/4} \tag{11}$$

Expressing E_n in terms of K_n and replacing l_n in the dimensionless form of Eqn. (9) with its value from Eqn. (10) and integrating, we find :

$$Z_n = \frac{4}{X} - 4 \tag{12}$$

The constant, —4, has been determined from the boundary condition :

$$K_n\,|_{Z_n = 0} = 0 \tag{12}$$

On the basis of Eqns. (11) and (12), Eqn. (4) may be written as :

$$\frac{du_n}{dX} = -4\,/\,X^2\,(1-X^4) \tag{13}$$

Integrating Eqn. (13) with respect to X we get the following dimensionless form of velocity:

$$u_n = \frac{4}{X} + 2\tan^{-1} X + \ln\frac{1-X}{1+X} + C \tag{14}$$

Eqns. (10) to (14) together with the values of the scaling parameters provide the following non-linear profiles :

$$K = ku_*\, L(1-X^4)$$

$$l = C^{1/4}\, kL\left(\frac{1-X^4}{X}\right)$$

$$\epsilon^{\frac{1}{2}} = \frac{u_*}{C^{1/4}}\, X$$

$$\theta = -\frac{H}{\rho\, c_p}\, \theta_n$$

$$u_n = \theta_n + C_1$$

$$= \frac{4}{X} + 2\tan^{-1} X + \ln\frac{1-X}{1+X} + C,$$

$$X = 4/(\, Z_n + 4\,) \tag{15}$$

It must be noticed that the derived profiles differ from Zilitinkevich profiles. This is mainly provided from the dependance of the profiles equations on the parameter :

$$X = (1-K_n)^{1/4}$$

which is given from the relation :

$$Z_n = \left(\frac{4}{X}\right) - 4.$$

Zilitinkevich corresponding parameter :

$$y = (\,1 - K_n)^{1/4}$$

is given from a completely different relation :

$$Z_n = \frac{2}{y} - \frac{2}{3}\, y^3 - \frac{4}{3}$$

3. Asymptotic behaviour of the model

The prediction of the asymptotic behaviour of the model is followed from the limits of Z_n under very stable, very unstable and neutral conditions. In the very stable limit (very unstable) $Z_n \to \infty$ $Z_n \to -\infty$). The value of X retained in this case is $X \to 4/Z_n$ as provided from equation (12) and $l_n \to 1/X$ in equation (10). This provides the following asymptotic form of the equations of the model :

$$K = ku_* L$$

$$l = \frac{C^{1/4}}{4}\, kZ$$

$$\epsilon^{\frac{1}{2}} = \frac{4}{C^{1/4}}\, u_*\, \frac{L}{Z}$$

$$u = \frac{u_*}{k}\, (\, \frac{Z}{L} + C)$$

$$\theta = -\frac{H}{\rho\, c_p}\left[\frac{Z}{L} + (\,C - C_1\,)\right] \tag{16}$$

Determination of the asymptotic behaviour under the neutral conditions, $Z_n \to 0$, may be derived by expanding in series the expression of Z_n arised from Eqns. (11) and (12). This gives $K_n = Z_n\,(1 - \frac{5}{8}\, Z_n + \ldots.)$ which implies that $K = ku_* Z$ under neutral condition. Replacing K in Eqns. (4) and (5) with its value and integrating with respect to Z we get the logarithmic form of wind (and temperature) profile.

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