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A NOTE ON DIURNAL MAXIMUM OF ROOM TEMPERATURE

Atmospheric temperature at screen level attains its maximum sometime generally between 2 & 3 p.m., when decaying insolation rate is balanced by growing long wave radiation of the earth. But the air inside a closed room where ventilation is poor avoiding, mixing will attain its diurnal temperature maximum much later, may be five to six hours after the time of screen temperature maximum.

This fact is the result of heat conduction in the form of wave motion through the walls and roof. As it will be shown in the next section the velocity of such wave motion is very low and the heat from outside takes several hours to enter the room by conduction. The phenomenon is particularly significant during summer when the temperature gradient between outer and inner sides of the wall is greater than that in winter.

2. *Mathematical derivations* — The process of heat motion through a wall may be considered as rectilinear damped wave propagation where the heat gained by every particle is spent in (1) increasing the temperature of the particle, (2) conducting to next particle and (3) radiating heat by that particle.

If we consider depth of the wall along X-axis such that outer side (warmer end) is $x=0$, the rectilinear wavemotion of heat is governed by the equation :

$$\sigma \frac{d^2T}{dx^2} = \frac{dT}{dt} \tag{1}$$

provided that the heat loss due to radiation is negligible.

where, dT/dx = Temperature gradient,

dT/dt = Rate of temperature increase with time

σ = Thermal diffusivity defined as $\dfrac{\text{Coefficient of thermal conductivity}}{\text{Thermal capacity per unit volume}}$

The change in temperature can be assumed simple harmonic with respect to time owing to diurnal fluctuation in atmospheric heat balance. Therefore, the solution of Eqn. (1) takes the form :

$$T = T_m \, e^{-cx} \sin\left(\omega t - \frac{2\pi x}{\lambda}\right) \tag{2}$$

where ω is the angular velocity of the wave motion, T_m is the maximum value of T, c is the damping coefficient and λ is the wave length.

Substituting values of $d^2 T/dx^2$ and dT/dt from Eqn. (2) in the differential Eqn. (1) and comparing the coefficient we get :

$$\omega = \frac{8\pi^2\sigma}{\lambda^2} \tag{3}$$

If t_0 is the time period of wave motion, then

$$t_0 = \frac{2\pi}{\omega} = \frac{\lambda^2}{4\pi\sigma} \tag{4}$$

$$\therefore \lambda = 2\sqrt{\pi\sigma t_0} \tag{5}$$

and the velocity of heat is

$$v = \frac{\lambda}{t_0} = 2\sqrt{\frac{\pi\sigma}{t_0}} \tag{6}$$

According to the definition

$$\sigma = \frac{k}{\rho s} \tag{7}$$

where k is the thermal conductivity, ρ is density and s is the specific heat of the wall. These constant can be known by experiments.

If variables are measured in C.G.S. units then from Eqns. (6) and (7) it can easily be derived that the time taken by heat flux to cross a wall of thickness d cm will be :

$$t = \frac{d}{2}\sqrt{\frac{t_0 \rho s}{\pi k}} \text{ seconds} \tag{8}$$

TABLE 1

Material	ρ (gm/cm³)	K (cal cm⁻³ sec⁻¹/°C)	s	$\sqrt{\sigma}$	t (min)
Quartz	2.630	.0200	.174	4.7	6.5
Marble	2.700	.0059	.210	9.9	13.7
Lime stone	2.730	.0045	.216	11.3	16.6
Clay	2.580	.0025	.197	14.3	19.8
Sand stone	2.650	.0041	.220	11.4	15.8
Coal	1.180	.0009	.225	16.1	22.2

For a diurnal wave :

$$t_0 = 24 \times 60 \times 60 \text{ seconds}$$

$$\therefore\ t = 82.9\, d \sqrt{\frac{\rho s}{k}} \qquad (9)$$

3. *Numerical Illustration* — Numerical values of σ and the time t (in minutes) taken to by heat wave to cross through 1 cm of a few building materials are illustrated in Table 1. The specific heat (s) of these materials are variable with temperature, the nature of variablility is not definitely known. The values of s considered here are for the temperature ranges, which are generally observed in diurnal variation of temperatures.

References

Banerji, R. C. and Upadhyay, D. S., 1972, *Mausam Vigyan.*

Lee Frederick, W., 1969, *Geophysical prospecting* published in *Mining Engineering handbook Vol. I.*

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