A cyclonic storm wave model for the Bay of Bengal and the Arabian Sea

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ABSTRACT. A cyclonic storm wave model is developed to compute maximum significant wave heights in deep waters in the Bay of Bengal and the Arabian Sea. Surface wind field required for such computation is simulated by a cyclonic storm model (Basu and Ghosh 1987) in which balance of all forces, including friction, is assumed. Bretschneider's (1972) hurricane wave model is used for computation of maximum significant wave heights. Pre-computed wave-prediction nomogram is evolved for operational use. Computations verify well with observations recorded on research vessels and oil rigs.

1. Introduction

The combined effect of winds and waves creates hazards to ships. When winds are strong and waves high, a vessel's safety is jeopardised and therefore, forewarnings of such phenomena are essential.

There are many empirical but useful forecast models for wave parameters which are related to surface wind speed and fetch. The model equations are developed basically by working out simple regressions based on observations. However, these models cannot be used to compute wave fields associated with tropical cyclones because of some special characteristics of the wind fields associated with them. These characteristics are extreme wind speed in wall cloud zones, relatively small dimensions resulting in small fetch and the motion of the cyclone. Consequently, a number of attempts have been made in computing wave fields generated by cyclones.

Francis Jr. (1944) was the first to present a method for forecasting cyclone waves. Bretschneider (1959) developed a formula for determining the maximum significant wave height and the period. Iijima et al. (1968) proposed a formula for typhoon wave field, including the maximum value for the significant waves. Bretschneider (1972) made improvement on his earlier paper (1959), and evolved a new formula based on additional data and experience. Utilising a few observations recorded on the oil rigs in Bombay High, Mukherjee and Sivaramakrishnan (1977, 1980) have put forward an empirical model for the distribution of winds and waves generated by a cyclonic storm in the Arabian Sea. Dextier (1983) reviewed wave forecast models, now in use in various parts of the globe. According to him Bretschneider's model is simple, easy to apply and useful. Bretschneider (1972) used a gradient wind formulation given by Graham and Nunn (1959) and obtained a maximum sustained wind at a height of 10 m above the surface by reducing the maximum gradient wind by a factor of 0.865 and by adding to it half the cyclone transmission speed.

But the gradient wind does not take into consideration the effect of the frictional force, which makes winds spiral inward to a cyclone centre, as observations indicate. As the wind is the most important parameter for wave generation, we use a cyclone wind model (Basu and Ghosh 1987) which takes into consideration balance of four forces — the pressure gradient, centrifugal, Coriolis and frictional — acting on a tropical cyclone. The wind model also uses a cyclone pressure profile which is evolved from observations of surface pressure observations, associated with a few past cyclonic storms in the Bay of Bengal. Bretschneider's (1972) wave model, is however, used in the present paper.

2. Data and method of analysis

Accurate measurement of wave is understandably difficult. This has resulted in inadequate wave data in our archives. Verification of the results of any model has a certain degree of uncertainty. In the present study
Fig. 1. Curve C as a function of $f(R_m/V_{G, max})$
(After Bretschneider 1972)

Fig. 2. Nomogram for computation of maximum significant wave height

**TABLE 1**

Computed and recorded observed values of wind speed and wave height

<table>
<thead>
<tr>
<th>$\Delta p$ (hPa)</th>
<th>Storm speed (km/hr)</th>
<th>Maximum sustained wind speed (kt)</th>
<th>Difference ($d)-(3)$</th>
<th>Error (%)</th>
<th>Wave height (metres)</th>
<th>Difference ($8)-(7)$</th>
<th>Error ($%_p$)</th>
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reliable actual wind and wave data recorded on oil rigs, dedicated scientific vessels and other ships are used. Observations from Indo-Soviet Monsoon Experiment during 1973 and Monex-1979 are reasonably reliable. In addition, ships' observations during the May 1975 Arabian Sea cyclone and observations made by three oil rigs in Bombay High area off the west coast of India during the June 1976 cyclone in the Arabian Sea, are extensively used. Pressure drops in cyclones which are comparatively numerous over the Indian Ocean are used to compute the maximum sustained wind. These are compared with observed data from which significant wave heights are computed. In addition, thirty-five ships' observations recorded by Mishra and Gupta (1976) for cyclonic storms in Indian seas are used.

2.1. Computation of maximum sustained wind

Under the assumption of balance of the forces (mentioned in Sec. 1), we get the following equations (Myers and Malkin 1961):

\[ \frac{1}{r} \frac{\partial p}{\partial r} \sin \beta + \frac{1}{r} \frac{\partial p}{\partial \theta} \cos \beta - k_r V_r^2 = 0 \]  
(1)

\[ \frac{1}{r} \frac{\partial p}{\partial r} \cos \beta - \frac{1}{r} \frac{\partial p}{\partial \theta} \sin \beta - f V_r - \frac{V_r}{r} \cos \beta - k_r V_r^2 + V_c \frac{V_c}{r} \sin \theta = 0 \]  
(2)

where \( p \) is the density of air, \( \theta \) is the azimuth angle, positive when measured clockwise from the direction of movement of the storm, \( \beta \) is the the deflection angle, clockwise from the direction of cyclone movement, \( V \) is the wind speed, \( f \) is the Coriolis parameter, \( r \) is the radius vector of a point from the cyclone centre, positive outward, \( V_c \) is the speed of propagation of the cyclone, \( k_r \) and \( k_c \) are empirical constants relating tangential and normal components of friction to square of \( V \). The above two equations involve three unknowns, so we need to know the distribution of one more parameter in a cyclonic storm field. The sustained wind is difficult to measure or estimate due to presence of thunderstorms/tornadoes. On the other hand, the central pressure which is subject to less random fluctuations and is easier to measure, is used in this study. The equation governing the distribution of pressure in a cyclonic storm field (Basu and Ghosh 1987) is used as below:

\[ \frac{\partial p}{\partial r} = \left[ p_p - p_e \right] \left( \frac{k-1}{R} \right) \left( \frac{r}{R} \right)^{k-1} e^{-\left( \frac{R}{k} \right) \left( \frac{r}{R} \right)^{k}} \]  
(3)

where \( p_p, p_e \) are respectively the peripheral and the central pressure, \( k \) is an empirical constant determined from pressure distributions in some real cyclonic storms formed over the Bay of Bengal, \( r \) and \( R \) being respectively the radial distance from the cyclone centre of any point in the field and the radius of maximum pressure gradient. The value of \( R, i.e., \) the radius of maximum pressure gradient is related to the radius of maximum winds, which can be estimated from location of wall clouds (Gentry 1973), obtainable from satellite imagery or radar observations. The pressure drop can be estimated by analysis of satellite imagery of cyclonic storms and related pressure drops (Mishra and Gupta 1976).

Solutions of Eqns. (1), (2) and (3) give values of \( V \) and \( \beta \) at every grid point in a cyclonic storm field for predetermined values of \( k_r, k_c \) and \( k \) when \( V_c \) is known. \( V_{mx} \), the maximum wind, is thus computed. The values of \( k_r, k_c \) and \( k \) determined by numerical experiment are taken as 0.0044 km\(^{-1}\), 0.0040 km\(^{-1}\) and 1.5 respectively.

2.2. Computation of maximum significant wave height

By using data for 51 hurricanes, Bretschneider (1972) has evolved the following equations for computation of maximum significant wave height associated with a tropical cyclone:

\[ V_{gmx} = K_B \left( \Delta p \right)^{1/2} - 0.27 f R_m \]  
(4)

\[ V_{mz} = 0.865 V_{gmx} + 0.5 V_c \]  
(5)

\[ H_{max} = C \left( R_m \Delta p \right)^{1/4} \left[ 1 + 0.5 \frac{V_c}{V_{mx}} \right] \]  
(6)

where \( V_{gmx} \) is the maximum gradient wind speed, \( H_{max} \) is the maximum significant wave height, \( \Delta p \) is the pressure drop, \( R_m \) is the radius of maximum wind and \( C \) is an empirical function of \( f(R_m/V_{gmx}) \) as shown in Fig. 1 and \( K_B \) is a constant. Bretschneider used a value of 21.0 for \( K_B \).

In the present study for computation of \( H_{max} \) Eqn. (6) is used but instead of the gradient wind used by Bretschneider, \( V_{mx} \) as obtained in the previous section is used. This equation is chosen because of its simplicity and wide use in operational forecasting.

3. Discussion of computations

3.1. Wind

Table 1 shows the results of the model winds. The actual wind observations are also given. Estimated values of pressure drop \( \Delta p \), in hPa and storm speed (km/hr) are given in columns 1 and 2. Maximum sustained wind speed (kt) computed by the the model with the corresponding observed values are given in columns 3 and 4 respectively. The difference between the computed and observed values and the percentage error are given inCols. 5 and 6 respectively. The root mean square error works out to 9.1 kt and the mean error is 17.9 per cent for the present model.

3.2. Waves

As stated in Sec. 2.2 maximum significant wave heights are computed from Eqn. (6) by using \( V_{mx} \) as obtained in Sec. 2.1. Table 1 also presents the values of wave heights computed by the model (Col. 7). Comparing this with column 8 which gives observed maximum significant wave heights, we see that they agree quite well. Column 9 presents the difference between observed and computed values and Col. 10 gives the percentage errors. Root mean square error for the present model works out to 1.1 m and the mean error works out to 8.3%.

3.2.1. Nomogram for maximum significant wave height

For operational use wave-prediction diagrams are desirable to the field scientists in view of non-availability of computing facilities and pressure on available time. Moreover, such a diagram helps in quick computation when cyclonic storm parameters change rapidly. With this object in view a nomogram is constructed for wave
prediction. With the help of this nomogram (Fig. 2) one can obtain the height of maximum significant wave height in a storm. In Fig. 2 the pressure drop and maximum significant wave height are along the abscissa and the ordinate respectively. There are two sets of curves A and B corresponding to $R$ (radius of maximum pressure gradient) equal to 15 and 20 km respectively. The scales for curves A and B are different and as indicated in the diagram. There are three curves in each of A and B corresponding to three different speeds of propagation of a storm, viz., 10, 15 and 30 km/hr.

4. Conclusions

From the above discussions one may arrive at the following conclusions:

(i) If pressure drop and radius of maximum pressure gradient are accurately known, maximum significant wave heights can be computed objectively within 90 percent of accuracy.

(ii) Speed of the storm being a sensitive parameter for computing wave heights, this value also needs to be estimated accurately.

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