The response of different wind-stress forcings on the surges along the east coast of India

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ABSTRACT. Experiments have been performed with a numerical storm surge prediction model (1981) to simulate the surges generated by the 1977 Andhra cyclone by using various formulae representing the wind distribution in the cyclone. It is found that most of these formulations do not give a good estimate of the observed wind associated with the Andhra cyclone. Accordingly, the computed sea surface elevations along the east coast of India do not agree with the observations.

In the present note, a new formulation for the computation of the surface winds has been suggested. A comparison of the results by using the well-known formulae for the wind distributions has been made.

1. Introduction

Wind is the principal cause for storm surge generation, although the strength of the wind in a cyclone is related to the pressure drop in it with respect to its surroundings. Wind exerts tangential as well as normal stresses on the water underneath, the latter being negligibly small compared with the former. The tangential stress generates long water waves or storm waves (i.e., storm surges). The height of the wave (or the surge height) depends on, among other things, the strength of the wind. The greater the wind speed, the higher is the surge peak.

So far no satisfactory theory exists on which a computation of the surface winds can be based. Variations in the tangential wind component with radial distance from the storm centre may be inferred from the pressure field. This gives good agreement with the occasionally observed winds in the region within the radius of maximum winds. In this central region the cyclostrophic wind or the gradient wind is a fairly good estimate of the actual wind.

A number of empirically based formulae for computation of the surface winds have been used by several workers (Jelesnianski, 1965, 1972; Isogaki, 1970; Das et al. 1974; Johns and Ali, 1980; etc) in their storm surge prediction models. However, it is found that most of these formulations do not give a good estimate of the winds at greater distances from the centre of the tropical cyclones. Accordingly, the surges generated by these winds may not agree with the observed sea surface elevations.

In the present note, a comparison of the winds computed by using various well-known formulae and their contribution to the surge induced by 1977 Andhra cyclone has been made. The track of the storm is shown in Fig. 1. An attempt is made to suggest an empirical formula which provides a better estimate of occasionally observed winds in the central region as well as at larger distances from the centre of the tropical cyclone.

The experiments suggest that the winds in the central region within the radius of maximum wind do not make a significant contribution while far distant winds play an important role in the generation of surges.
2. Basic equations

The basic hydrodynamic equations of continuity and momentum for the dynamical processes in a Bay are given by:

\[
\frac{\partial \xi}{\partial t} + \frac{\partial}{\partial x} \left[ (\xi + h)u \right] + \frac{\partial}{\partial y} \left[ (\xi + h)v \right] = 0
\]

(1)

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} - f v = -g \frac{\partial \xi}{\partial x} + \frac{1}{(\xi + h)\rho} (F_o - F_b)
\]

(2)

\[
\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + f u = -g \frac{\partial \xi}{\partial y} + \frac{1}{(\xi + h)\rho} (G_b - G_b)
\]

(3)

where,

- \(x, y, z\) describe a system of cartesian coordinates with \(x\) taken positive eastward, \(y\) northward and \(z\) vertically upward
- \(t\) time
- \(\xi\) elevation of the sea surface from its undisturbed state
- \(h\) depth of the sea bed
- \(u, v\) depth averaged zonal and meridional component of velocity
- \(f\) Coriolis parameter
- \(g\) acceleration due to gravity
- \(\rho\) density of the sea water
- \(F_o, G_b, x\) and \(y\) components of wind stress
- \(F_b, G_b, x\) and \(y\) components of bottom stress

Following Johns et al. (1981), Eqns. (1)-(3) are transformed in the curvilinear coordinate system and are written in the flux form as:

\[
\frac{\partial}{\partial t} \left[ \frac{b(y)}{\xi} \xi \right] + \frac{\partial}{\partial \xi} \left[ b(\xi + h)u \right] + \frac{\partial v}{\partial y} = 0
\]

(4)

\[
\frac{\partial u}{\partial t} + \frac{\partial}{\partial \xi} \left( U\xi \right) + \frac{\partial}{\partial y} (v\xi) - f \frac{\partial \xi}{\partial t} = -g \frac{\partial \xi}{\partial x}
\]

(5)

\[
\frac{\partial u}{\partial t} + \frac{\partial}{\partial \xi} \left( U\frac{\partial \xi}{\partial y} \right) + \frac{\partial}{\partial y} \left( v \frac{\partial \xi}{\partial y} \right) + f \frac{\partial \xi}{\partial t} = -g(\xi + h)
\]

(6)

where,

- \(b(y) = b_2(y) - b_1(y)\) is the breadth of the Bay,
- \(x = b_1(y)\) and \(x = b_2(y)\) being the western and eastern coastal boundaries
- \(\xi = \frac{x - b_1(y)}{b(y)}\), the transformed \(x\)-coordinate
- \(U = \frac{1}{b(y)} \{u - (b_1' - \xi b_2') v\}\)
- \(\xi' = b_1(\xi + h) u\)
- \(\xi'' = b_1(\xi + h) v\)

In the above equations, the bottom stress components are parameterised by a conventional quadratic law:

\[
F_b = \rho c_f u (u^2 + v^2)^{\frac{1}{2}}
\]

\[
G_b = \rho c_f v (u^2 + v^2)^{\frac{1}{2}}
\]
Eqns. (4)-(6) are solved subject to the following initial and boundary conditions (Johns et al. 1981):

\[ \varepsilon = \frac{\partial z}{\partial t} = U = \xi = 0 \quad \text{for} \quad t \leq 0 \]  
(7)

\[ U = 0 \quad \text{at} \quad \xi = 0 \quad \text{and} \quad \xi = 1 \]  
(8)

\[ \frac{\partial}{\partial y} \left( \frac{g}{h} \right) b (L + h) \xi = 0 \quad \text{at} \quad y = 0 \]  
(9)

\[ \frac{\partial}{\partial y} z = 0 \quad \text{at} \quad y = L \quad \text{(northern boundary of the Bay)} \]  
(10)

For the solution of Eqns. (4)-(6) subject to conditions (7)-(10), a conditionally stable explicit finite difference scheme with a staggered grid is used. The details of numerical scheme is given in Johns et al. (1981).

3. Wind stress functions

In the predictive Eqns. (5) and (6) the wind stress components are used as forcing functions. The surge is generated by an idealised circular storm, symmetric in wind speed about its centre, tracking across the analysis area with a constant speed. It is desirable to formulate wind stresses that act on a sea surface in analytic form which approximates the passage of a tropical cyclone.

A number of numerical models specify the pressure field by one of the following expressions:

\[ p(r) = p(\infty) - \frac{\Delta p}{1 + (r/R)^{3/4}} \quad \text{(Isozaki 1970)} \]  
(11)

\[ p(r) = 1010 - \frac{\Delta p}{1 + (r/R)^{3/4}} \quad \text{(Das et al. 1974)} \]  
(12)

\[ p(r) = p(\infty) - \Delta p e^{-r/R} \quad \text{(Johns and Ali 1980)} \]  
(13)

where, \( p(r) \) and \( p(\infty) \) represent sea-level pressure at \( r \) and at the cyclone periphery, respectively, \( R \) is the radius of maximum wind and \( \Delta p \) is the pressure drop.

The wind distribution in the cyclone is then computed using the cyclostrophic or gradient wind formulae given by:

\[ V^2 = 4\nu_m \left( \frac{(r/R)^3}{1 + (r/R)^3} \right) \]  
(14)

\[ V = \frac{f r}{2} + \left( \frac{f^2 r^2}{4} + \frac{r^2}{p_a} \right)^{1/2} \]  
(15)

where \( p_a \) is the density of the air and \( V_m \) is the maximum wind at \( R \). By Eqns. (12) and (14) the maximum wind and the pressure drop may be related by:

\[ V_m = c(\Delta p)^{1/4} \]

where \( c \) is a numerical constant.

In some other cyclone models the wind profile is specified by

\[ V = \nu_m (r/R)^{3/2}, \quad 0 < r < R \]

\[ V = \nu_m (r/R)^{1/2}, \quad r > R \quad \text{(Jelesnianski 1965)} \]  
(16)

\[ V = \nu_m \frac{2R}{r^2}, \quad \text{(Jelesnianski 1972)} \]  
(17)
TABLE 1

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Computations show that most of the above well known formulae do not produce a satisfactory estimate of the actual wind associated with tropical cyclones in the Bay of Bengal. Accordingly, an empirical formula which gives a better estimate of the winds inside and outside the central core of the cyclone has been suggested:

\[
V = V_m e^{(r-R)/\alpha}, \quad 0 \leq r \leq R
\]

\[
V = V_m e^{(r-R)/\beta}, \quad r \geq R
\]

(18)

where \( \alpha \) and \( \beta \) are numerical constants having the dimensions of length. This new wind formulation is in good agreement with the observed wind fields during 1977 Andhra cyclone for \( \alpha=15 \) km and \( \beta=240 \) km.

The surface stress components \( (F_s, G_s) \) due to the action of wind are calculated by a conventional quadratic law with an uniform friction coefficient of \( 2.8 \times 10^{-3} \).

4. Results and discussions

Using the model developed by Johns et al. (1981) the surge heights have been computed for the 1977 Andhra cyclone by taking different types of wind formulation given by Eqns. (14)—(18). Following the reports from India Meteorological Department, the numerical experiments are performed by taking \( \Delta p = 65 \) mb, \( R = 80 \) km and \( V_m = 70 \) m/s. The analysis area is the same as in Johns et al. (1981).

Table 1 gives an account of the winds computed by using different formulae (14)—(18) and the observed winds associated with 1977 Andhra cyclone. It may be seen that the winds computed by our formulation (Eqn. 18) are in good agreement with the observations.

Figs. 2(a-c) and 3(a-c) give the time variation of sea surface elevation at five stations (Contai, Visakhapatnam, Divi Island, Kavali and Pondicherry) along the east coast of India as computed by using the wind forcing given by Eqns. (14)—(18), respectively.

Considering first the Andhra region (consisting of Kavali and Divi Island) we find that in all cases of wind forcings the maximum surge occurs at Kavali. However, the peak surge heights vary from 6 m (Fig. 3a) to 2.4 m (Fig. 2b). This variation in the sea surface elevations may be attributed to the strength of the winds estimated by different formulae. It may be seen from Table 1 that the winds computed by using Eqn. 17 are slightly overestimated while those computed from Eqn. (15) are much underestimated compared to observations in the
strongest wind region of the tropical cyclone. The qualitative behaviour of the predicted surges at Divi Island is the same as at Kavali.

In the northeast region of Andhra Pradesh, at Visakhapatnam, the maximum predicted surge elevation vary between 3.7 m (Fig. 2c) and 0.5 m (Fig. 2b). The response predicted at Contai in West Bengal by using wind formula (16) is anomalous (Fig. 2c) while those computed from other wind formulations appear to be reasonable. The high responses shown in Fig. 2(c) at far distant places (Visakhapatnam and Contai) to the right of the position of landfall are again the result of very strong winds at large distances from the centre of the cyclone (200-500 km).

At Pondicherry which is about 450 km to the south of the place of landfall, the surge response vary between 1.3 m (Fig. 2e) and 0.4 m (Fig. 3b).

It may be inferred from the above results that the winds at larger distances from the centre of the cyclone significantly affect the sea surface elevations. As the new wind formulation (Eqn. 18) suggested in this note gives a good estimate of the observed large distant winds (Table 1), the predicted surges at all the five stations along the east coast of India are reasonably good (Fig. 3b).

In order to see the role of winds in the central region of the cyclone an experiment has been done with the central wind represented by the first relation of Eqn. (16) and the far distance wind by the second relation of Eqn. (18). This provides very strong winds in the central region as compared to our new formulation (Table 1). The results of this experiment are shown in Fig. 3(c). A comparison of Figs. 3 (b & c) shows insignificant difference in the surge responses. This suggests that the sea surface elevations are not very sensitive to central core winds.

5. Concluding remarks

Using six different surface wind formulations we have simulated the surge generated by 1977 Andhra cyclone. On the basis of the above results, following general conclusions may be drawn:

(i) In order to have an accurate prediction of the surges, the computed winds associated with a tropical cyclone should agree with the observed wind fields as far as possible.

(ii) The winds in the central region do not contribute significantly while the far distant winds play an important role in the generation of surges along the coast.

(iii) Further study is required for the improvement in the theory on which the computation of the winds associated with a tropical cyclone in the Bay of Bengal may be used.
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References


