Simulation of north Indian Ocean circulation using a simple barotropic model and some sensitivity studies

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ABSTRACT. A non-divergent barotropic model has been formulated on the basis of splitting up method and used to study the circulation in the north Indian Ocean (1-26°N, 46-99°E). The circulation was simulated for summer and winter seasons separately. It is found that the model simulated the summer and winter circulations satisfactorily. It is also found that the meridional component of wind stress is dominant over the zonal component in shaping the Somali current. Some sensitivity studies were also carried out and the results indicate the importance of wind stress curl.

Key words — Ocean circulation, Simulation, Splitting, Simple barotropic.

1. Introduction

Several mathematical modelling works were carried out in 50's and early 60's using barotropic models to study the general circulation of oceans (Stommel 1958, Munk 1950, Bryan 1963). In the next two decades, with increase in computational facilities and formulation of numerical techniques more advanced three dimensional models were attempted (Bryan 1969). Due to availability of sophisticated ocean general circulation models (OGCM) and ocean eddy resolving general circulation models (OEGCM) the simple barotropic modelling works were partially abandoned in the last decade. However, due to complexity and computational expenses involved in OGCM and OEGCM the simpler two dimensional models are being reconsidered to study specific problems, e.g., the reduced gravity transport model (Luther and O'Brien 1985, Simmons et al. 1988).

In the present study we have used splitting up method to formulate a simple non-divergent barotropic model that is used to simulate the major features of the north Indian Ocean circulation. The circulations for the summer and winter seasons are simulated starting from a state of rest using the surface wind stress of each season as forcing. Some sensitivity studies were also carried out with different idealised wind stress of summer case.

2. The model

The model is based on the following differential equations (under hydrostatic balance and rigid lid approximation), considered on a two dimensional bounded region Q with a lateral surface σ. The model ocean is homogeneous with respect to density and assumed to be confined between two horizontal plates:

\[
\begin{align*}
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} &= -f v + \frac{1}{\rho_0} \frac{\partial P}{\partial x} + \frac{1}{\rho_0} \frac{\tau_{yx}}{H} \\
\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} &= -f u + \frac{1}{\rho_0} \frac{\partial P}{\partial y} + \frac{1}{\rho_0} \frac{\tau_{yx}}{H} \\
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= 0
\end{align*}
\]
The $x$-coordinate is positive eastwards and $y$-coordinate is positive northwards. $(u, v)$ are the depth independent $(x, y)$ components of velocity. $A_f$ is the horizontal eddy viscosity coefficient, $(\tau_x, \tau_y)$ are the wind stress $(x, y)$ components applied as body force, $H$ is the depth of the model ocean, $f$ is the Coriolis parameter and $\rho_0$ is the sea water density $= 1.028$ gm/cm$^3$. The other notations carry their usual meanings. The boundary conditions along the lateral boundaries $\sigma$ is no slip :

$$u = v = 0$$

(2)

The equations are linearised over each time step. Then we can write the equations in operator form :

$$B \frac{\partial \Phi}{\partial t} + A_2 = F$$

with initial data at $t = 0$.

$$B_2 = BQ$$

(3)

where,

$$\begin{pmatrix}
\rho_0 & 0 & 0 \\
0 & \rho_0 & 0 \\
0 & 0 & P
\end{pmatrix}
\begin{pmatrix}
u \\
\tau_x \\
\tau_y
\end{pmatrix}
= \begin{pmatrix}
\rho_0G - \rho_0f \frac{\partial}{\partial x} \\
\phi G - \rho_0f \frac{\partial}{\partial y} \\
\frac{\partial}{\partial x} \frac{p}{\gamma} - \frac{\partial}{\partial y} \frac{p}{\gamma}
\end{pmatrix}
$$

$$\begin{pmatrix}
\tau_x[H] \\
\tau_y[H] \\
Q
\end{pmatrix}
= \begin{pmatrix}
u^0 \\
\gamma^0 \\
\rho^0
\end{pmatrix}
$$

(4)

We assume that the components $(\bar{u}, \bar{v})$ are known over each time interval. They satisfy the continuity equation and also satisfy the boundary condition :

$$\frac{\partial \bar{u}}{\partial x} = 0$$

(5)

where $\bar{u}$ is the normal component of velocity vector and superscript $j$ refers the time step.

Now it is taken that the solution of the Eqns. (3) & (4) belongs to Hilbert subspace (Marchuk 1975). We introduce the scalar product with the relation :

$$(a, b) = \sum_{i=1}^{3} \int_{\Omega} a_i b_i d \Omega$$

So it can be verified that $(A_1, \varphi) > 0$. The model equations are solved using splitting of the main processes into two simpler ones based on physical processes (Marchuk 1972, 1975):

(i) Advection diffusion processes, and

(ii) Adaptation processes.

2.1. Splitting of the problem

We split the operator $A = A_1 + A_2$ on the basis of physical processes, where,

$$A_1 = \begin{pmatrix}
\rho_0G & 0 & 0 \\
0 & \rho_0G & 0 \\
0 & 0 & 0
\end{pmatrix}$$

$$A_2 = \begin{pmatrix}
0 & -\rho_0f & \frac{\partial}{\partial x} \\
\rho_0f & 0 & \frac{\partial}{\partial y} \\
\rho_0f & 0 & \frac{\partial}{\partial x} \frac{p}{\gamma} - \frac{\partial}{\partial y} \frac{p}{\gamma}
\end{pmatrix}$$

(6)

then it can be verified that $(A_1, \varphi, \zeta) > 0$, $(A_2, \varphi, \zeta) = 0$ for all $\varphi \neq 0$ which is necessary for stability of the scheme.

Let the entire interval $0 \leq t \leq T$ be broken up into equal time intervals $t_{i-1} \leq t \leq t_i$ and $t_i < t < t_{i+1}$ of width $t_{i} - t_{i-1} = \Delta t$ and over every expanded time interval $t_{i-1} \leq t \leq t_{i+1}$ we use the two cycle splitting up method for second order accuracy in time as a one cycle method gives only first order accuracy in time for non-commutative operators.

Solution of Eqn. (3) in the entire time interval $t_{i-1} \leq t \leq t_{i+1}$ is obtained in three stages. The first and last stages consist of advection-diffusion processes whereas second stage consists of adaptation processes.

Let $\varphi^{j-1}$ be the initial value of Eqn. (3) at time $t_{i-1}$, i.e., $\varphi^{j-1} = \varphi^{j-1}$. Then in time interval $t_{i-1} \leq t \leq t_i$, we have

$$B \frac{\partial \varphi^{j-1}}{\partial t} + A_2 \varphi^{j-1} = F$$

with initial condition

$$B \varphi^{j-1} = B \varphi^{j-1}$$

similarly in time interval $t_{i-1} \leq t \leq t_{i+1}$ (two consecutive steps are combined) we have :

$$B \frac{\partial \varphi}{\partial t} + A_2 \varphi = 0$$

with initial condition

$$B \varphi = B \varphi$$

and in time step $t_{i} < t < t_{i+1}$ we have :

$$B \frac{\partial \varphi}{\partial t} + A_2 \varphi = F$$

with initial condition

$$B \varphi = B \varphi^{j-1}$$

(7)

We note that in Eqn. (3) $\bar{u}, \bar{v}$ carry the same values over the entire interval $t_{i-1} \leq t \leq t_{i+1}$.

2.2. Difference approximation

The principal grid points in the horizontal domain have integer indices $(k, l)$ i.e., $x_k, y_l$ and the subsidiary points, $x_{k+1/2}, y_{l+1/2}$ are the mid points of the basic intervals.

2.2.1. Advection-diffusion problem

We now consider $u_{k+l}$ and $v_{k+l}$ satisfy the difference analog of the continuity equation :

$$\frac{u_{k+l} - u_{k-1/2,l}}{\Delta x} + \frac{v_{k+l} - v_{k,l+1/2}}{\Delta y} = 0$$

(8)

and the operator $G = \sum_{i=1}^{2} G_i$ is in the difference form.
where,
\[ G_A^k(\varphi) = \left( u_{k+1/2}^h \varphi_{k+1/2}^h - u_{k-1/2}^h \varphi_{k-1/2}^h \right) / 2\Delta x \]
\[-A_h(\varphi_{k+1/2}^h - 2\varphi_k^h + \varphi_{k-1/2}^h) / \Delta x^2 \]
\[ G_b^k(\varphi) = \left( v_{k+1/2}^h \varphi_{k+1/2}^h - v_{k-1/2}^h \varphi_{k-1/2}^h \right) / 2\Delta y \]
\[-A_h(\varphi_{k+1/2}^h - 2\varphi_k^h + \varphi_{k-1/2}^h) / \Delta y^2 \]
(9)

\[ \varphi \text{ is } u \text{ or } v. \]

It may be noted \( G_A^k \) is approximated with second order accuracy in spatial coordinates (for equal grid length). If we introduce a scalar product in the Hilbert space of grid functions \((a_I, b_I) \in G(G_h) \) then
\[ (a_I, b_I) = \sum_{k=1}^{n_k} \sum_{l=1}^{n_l} a_{k,l}^I b_{k,l}^I \Delta x \Delta y \]
(10)

It can be verified \((G_A^k, \varphi, \varphi) \geq 0, \quad i = 1, 2\)

Taking into account Eqn. (9) the system of differential equation approximating the first and last of Eqn. (7) take the following form in operator notation:
\[ B \frac{\partial \varphi}{\partial t} + A_h \varphi = F, \quad i = 1, 2 \]
(11)

It follows from the properties of the operator \( G_A^k \) that
\[ (A_h \varphi, \varphi, \varphi) \geq 0. \]

Eqn. (11) is solved over each interval \( t_{i-1} < t < t_i \) using Crank-Nicholson scheme with adoption of the two cyclic splitting method. As a result we arrive at simplest one dimensional problems which can be solved by factorisation method. Similarly the third equation of Eqn. (7) is solved over time interval \( t_i < t \leq t_{i+1} \) after obtaining the solution of the second equation of Eqn. (7) which is discussed in the following sub-section:

2.2.2. Adaptation problem

In this stage momentum equations in finite difference form are obtained and pressure gradient term is eliminated after getting two simultaneous equations in term of \( \Psi \) and the resultant equation is applied at sub-point \((k+1/2, l+1/2)\).

To solve the second Eqn. of Eqn. (7) we introduce the following difference operators in place of \( \partial / \partial x \) and \( \partial / \partial y \)
\[ \nabla_x^+ \varphi_k = 1/\Delta x \left( \varphi_{k+1} - \varphi_k \right), \]
\[ \nabla_x^- \varphi_k = 1/\Delta x \left( \varphi_{k-1} - \varphi_k \right), \]
\[ \nabla_y^+ \varphi_k = 1/\Delta y \left( \varphi_{k+1} - \varphi_k \right), \]
\[ \nabla_y^- \varphi_k = 1/\Delta y \left( \varphi_{k-1} - \varphi_k \right). \]

The equation is solved using Crank-Nicholson scheme, introducing stream function
\[ u_{t+1/2}^{e+1} + u_{t-1/2}^{e-1} = -\nabla_y^+ \Psi, \quad \text{and} \quad v_{t+1/2}^{e+1} + v_{t-1/2}^{e-1} = -\nabla_x^+ \Psi, \]

Then the solution reduces to an equation for the function
\[ \nabla_x^+ \Psi_{k+1/2}^e + \nabla_y^+ \Psi_{k+1/2}^e + \Psi_{k+1/2}^e = q_{k+1/2}^e, \]
with the condition \( \Psi_{k+1/2}^e = 0 \) on \( \partial \Omega_h^e \).

where,
\[ \Delta x_k \Psi_{k+1/2}^e = \frac{1}{2} \Delta x \left( \Psi_{k+1}^e - \Psi_{k-1}^e \right), \]
\[ \beta = \nabla_x^+ \Psi, \]
\[ q_{k+1/2}^e = \nabla_x^+ v_{k+1}^e - \nabla_y^+ u_{k+1}^e. \]

Eqs. (12) can be solved by relaxation method.

\[ u_{k+1/2}^e, v_{k+1/2}^e \]

are defined in the following approximation:
\[ u_{k+1/2}^e = \left( u_{k+1}^e + u_{k+1}^e \right)/2, \]
\[ v_{k+1/2}^e = \left( v_{k+1}^e + v_{k+1}^e \right)/2 \]

and they satisfy Eqn. (8). The scheme described is absolutely stable and energetically balanced (Marchuk 1975, Marchuk and Kordzadze 1986).

3. Results

Many model studies of the Indian Ocean have addressed the Somali current system (Cox 1970, 1979, 1981, Hurlburt & Thompson 1976, Leetma et al. 1982, Luther & O'Brien 1985, Simmons et al. 1988, McCreary & Kundu 1988). A striking feature of Somali current is that unlike other western boundary currents, it reverses its direction seasonally in response to change in the direction of prevailing winds over the region. During northern summer the winds over northwestern Indian Ocean are southwesterly which drive a strong poleward boundary current along the Somali coast. In the winter, the direction of the winds change to north easterly and a weak equatorward flowing boundary current develops along the Somali coast.

In the Bay of Bengal the currents are more or less in phase with the currents along the Africa coast but of lesser magnitude (Rao et al. 1989). However, there are not much attempts made to study the circulation in Bay of Bengal (Unnikrishnan & Bahubalayan 1991).

3.1. A case study

For model geometry we have considered the region 46°-99°E and 1°-25°N. Islands are not considered in this study. The model parameters used are \( A_H = 10^6 \) cm$^3$s$^{-1}$, \( H = 6 \times 10^9$ cm, \( \Delta x = 107$ km, \( \Delta y = 111$ km, \( \Delta t = 3$ hrs. The seasonal climatic wind stress for summer and winter seasons (Figs 1 (a&b)) are used to force the model (the data has been obtained under the Indo-USSR Integrated Long Term Programme of Cooperation in Science and Technology). The model is integrated from an initial condition of rest in both the cases. The depth parameter 60 m is chosen as with a realistic ocean depth the wind forcing as a body force will be unrealistically small and the 60 m depth represent an average upper layer depth above thermocline.
In both the cases Somali current spun up quite realistically flowing southward in winter and northward in summer [Figs. 2(a & b)] which can be compared with the observed surface circulation for January and July (Rao et al. 1989). The model reaches a quasi-steady state in about 30 days. Hence all the results are presented after 30 days of integration.

3.1.1. Winter circulation

The winter circulation [Fig. 2(a)] shows that the current off the Somali coast is flowing equatorward starting from 20°N. Between 18°N and 21°N the flow is zonal (westward) and turns southward along the boundary between 55°E and 60°E. The equatorward flowing Somali current meets the eastward flowing equatorial current between 2°N and 5°N. There exists a cyclonic gyre due to the recirculated Somali current at 7°N, 55°E. Another cyclonic gyre is seen near 11°N, 57°E. The flow in the interior is basically northwards through Sverdrup balance. The circulation is westward between 5°-8°N and 60°-80°E. The flow near the east coast of India is mainly southward that meets the westward flowing current near the tip of the Indian Peninsula. However, between 10°N & 15°N the flow is poleward along the coast. The interior flow is poleward through Sverdrup balance.

3.1.2. Summer circulation

During the summer monsoon the northward flowing Somali current starts from the southern boundary with a high meridional component in between 50°E and 52°E [Fig. 2(b)]. The magnitude of the current enhances with the addition of the recirculated southward flowing interior flow a part of which merges with Somali current between 5°N and 8°N. The other part of the southward flowing interior flow merges with the eastward flowing equatorial current. The poleward flowing Somali current leaves the boundary effectively between 18°N and 22°N. One branch of this zonal flow recirculates southwards and merge with the interior flow to complete an enormous clockwise circulation, embedded in this circulation there are eddies—one at 10° N and the other at 12° N. The general flow between 2°N and 5°N is eastward, but between 6°N and 8°N it is westward which does not agree very well with observation (Rao et al. 1989). South of 7°N in the general flow there exists at least four cyclonic eddies, the one between 70°E and 75°E is quite intense. The circulation in the Bay of Bengal is weaker compared to winter circulation. Along the east coast of India the flow is poleward and the interior flow is equatorward.

4. Discussion

4.1. Winter season

Due to the prevailing wind the Somali current flows equatorwards along the western boundary. The interior flow can be computed from a simple Sverdrup balance [if we take the vorticity equation of Eqn. (1) and neglect other terms the Sverdrup balance is determined

\[ \beta v = \frac{1}{\rho_0 H} \left[ k \cdot \nabla \times \tau \right] \]

So it can be seen from
4.2.1. Meridional wind stress forcing

The circulation features are almost similar to that of the circulation obtained with the actual wind stress forcing. For studying the western boundary currents which show a marked response to change in wind stress fields, we restrict our discussion to the two western boundaries: the Somali coast and the east coast of India.

The circulation feature indicates that the meridional component of wind stress is instrumental in shaping the Somali current and its associated gyre system, the magnitude of currents are higher than the currents obtained from the actual wind forcing (Figs. 3 (a&b)). The gyre is formed at nearly the same place in both the cases with less spatial extent in the case of meridional wind stress forcing.

The circulation obtained along east coast of India [Figs. 3 (a&b)] suggests that the meridional component of the wind stress also plays a dominant role in shaping the western boundary current. However, the magnitudes of the currents are more.

4.2.2. Zonal wind stress forcing

Unlike the meridional wind stress case the circulation obtained with the zonal wind stress forcing is weak and does not show the organized gyre system [Figs. 3 (c & d)]. Equatorward of 7° N the current is against the Somali current that may be reducing the magnitude of the current in actual wind forcing case. However, the general current direction poleward of 7° N is northward. But the currents along east coast of India are opposite to the currents obtained by meridional forcing. This may be the reason why the currents along east coast of India is not as strong as the currents along Somali coast.

The circulation features obtained by actual wind stress, the meridional component alone and the zonal component alone [Fig. 2(b), Figs. 3 (a-d)] suggests that the zonal component is dominant equatorward of 7° N and the meridional component is dominant poleward of 7° N.

We have not addressed here specifically the influence of local alongshore wind and the influence of offshore remote forcing but with above experiments we assume that the local wind stress is instrumental in shaping the Somali current, which is enhanced by the remote forcing away from the equator through reflected Rossby waves.

4.2.3. Sensitivity studies with idealized wind stress forcing

We have further investigated the influence of different idealized wind stress forcings on the circulation of the Arabian Sea.
Figs. 4 (c–d). Computed circulation with idealized wind forcing: (a) Actual summer wind stress up to approx. 500 km from the coast, (b) Curl free wind stress up to approx. 500 km from the coast, (c) Meridional wind stress with linear zone 1 drop (solid lines represent wind stress curl in $10^{-9}$ dynes/cm$^2$), and (d) Curl free wind stress through out the basin.

(i) Local alongshore wind stress forcing

In this experiment we have investigated the influence of local wind by considering the forcing up to nearly 500 km from the west coast. The results obtained with actual wind stress forcing, i.e., without any uniformity in the $x$ and $y$ directions, Fig. 4(a) shows two anticyclonic gyres along the western boundary in place of one huge gyre as in Figs. 3 (a & b). This may be due to the wind stress pattern we have chosen and the coastline. We get two patches of strong western boundary currents—one along the Somali coast and the other along the Arabian coast. The return flow is equally strong and more concentrated than in Figs. 3 (a & b) which may be attributed to the strong curl [Fig. 5(a)] arising 500 km away from the coast which gives rise to strong interior flow through Sverdrup balance. The flow eastward of 60° E is basically meridional which is possibly excited due to the edge in the wind stress. In the next experiment we have considered an idealized uniform (wind $\tau_{xx} = \tau_{yy} = 2$ dynes/cm$^2$) up to 500 km as
in the previous case. The circulation [Fig. 4(b)] in this case also found to be similar to the previous case but the magnitude of currents are stronger in the later case. This is consistent with the input. In the previous case wind stress is dominated by meridional component and stronger near the coast but in the later case there is a strong wind stress curl [Fig. 5(b)] after 500 km and a comparable zonal component giving rise to strong interior flow and hence a strong boundary current exists. When we shift the offshore curl to about 1000 km the boundary currents along the Somali coast weakens and the associated gyre becomes weak, however, the current along Arabian coast and the gyre are not affected which may be due to the geometry of the basin and coastline to the south of the gyre.

(ii) Meridional forcing with zonal linear drop

In this case we have used a meridional forcing of 2 dynes/cm² and linearly dropping it to zero at around 2000 km away from the coast. Note that, here the stress is not uniform along y-direction and the drop is rather smooth in x-direction and hence not so strong curl exists in the vicinity of the boundary currents and the gyres which resulted in a boundary current of lesser magnitude [Fig. 4(c)] and less organised gyres.

(iii) Curl free forcing

In this experiment we have considered a curl free uniform southwesterly wind stress \( \tau_{yx} = \tau_{yz} = 2 \) dynes/cm² and the results [Fig. 4(d)] clearly show the positive influence of the curl.

5. Conclusions

The non-divergent barotropic model formulated on the basis of splitting up method is found to be working satisfactorily in simulating the main features of the north Indian Ocean.

The role of the wind stress component is studied. The study indicate that the meridional component plays a dominant role in shaping the Somali current. The solution found to be affected by the strength of the curl, coastal configuration and the position of the curl from the coast. The performance of the model in simulating the currents near southern boundary is not satisfactory due to imposed artificial wall.

In our further studies, we will extend our southern boundary to further south which will allow us to study the southern hemispheric influence on Somali current and the equatorial currents. We will also replace the closed boundary conditions with suitable open boundary conditions. The scheme can effectively be used to formulate a three dimensional baroclinic ocean circulation model.

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References


