A turbulent convection theory of radar angels

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ABSTRACT. The problem of back-scattering from invisible atmospheric sources and the near zone concept put forward by Atlas are reviewed in relation to some angels observed at Bombay and New Delhi. It is seen that the near zone concept is not applicable to these cases. It is believed that a larger radar cross-section can be obtained in the far zone case by assuming a cylindrical model of the atmospheric eddies. Such eddies may occur in a region of turbulent convection. These air parcels consisting of homogeneous refractive medium can be treated as units for back-scattering treatment and may yield a considerably larger cross-section. A number of angels observed on PPI and REI are explained on this model. ‘Angel bands’ observed on REI at Delhi are found to be associated with moist layers at corresponding heights. Some particularly intense echoes observed on PPI at Bombay are attributed to the probable existence of a break in the inversion layer over the coastal region from which the radar waves are predominantly scattered towards sea and the corrugated sea surface in turn back scatters sufficient energy to reach the receiver.

1. Introduction

Radar echoes from apparently invisible sources have claimed considerable interest during the past few years. The source is in all probability the atmosphere though on the basis of the existing back-scatter theory, the gradient of refractive index needed to explain these echoes, particularly in the microwave region, far exceeds the largest observed so far. The main problem therefore, has been to investigate the possibility of obtaining the required back-scattering without having to assume phenomenally large refractive gradients. A particularly elegant attempt in this direction has been made by Atlas (1960) wherein the scattering surfaces are assumed to be sections of spheres with radii of curvature greater than the radar range so as to be near zone cases. The surfaces need be only large enough to include a small number of Fresnel zones. On the basis of this theory many of the observations (particularly those of Atlas and Plank) are explained with gradients which, although large, may reasonably be expected to occur in the atmosphere. However Atlas’ theory as he himself has remarked, is not able to account for many other observations. The angels explained by this theory are those observed either with vertically pointing or fixed horizontal beams at comparatively short ranges. At ranges of the order of 50 km, in order to be a near zone problem the spherical surface has to be essentially a plane. Further, for angles of incidence other than normal, the Fresnel zones will be ellipses rather than circles, with the eccentricity increasing as the deviation from normal increases. If the zones are inclined at large angles with the direction of propagation the amplitude of excitation will not be constant but vary markedly over a zone so that the contribution of successive zones cancelling and leaving that of the first Fresnel zone alone as the resultant intensity cannot be implicitly assumed. With the eccentricity of the zones increasing, contribution from the higher order zones, particularly along the minor axis of the ellipses would have to be considered.

With the angels observed on PPI, the difficulty becomes all the more acute. It is rather difficult to visualize large eddies or vapour sheaths with plane sides, as they have to be if they are to be treated as near zone cases at 50-km range. Again, except in association with sea breeze, there is no particular justification in assuming the vapour sheaths to be oriented always normal to the radar beam. A good number of angels observed on the PPI scope (Ligda and Bigler 1958; Bigler 1959; Rai 1959, 1960, 1961) cannot
possibly be treated as near zone sphere problems. One has to take recourse to the far zone treatment only.

The commonly assumed spherical model of atmospheric bubble is to be regarded incorrect in any fresh approach to the problem. If we can start with an altogether different geometrical shape to obtain a large radar cross-section, we may arrive at a reasonable solution, provided the assumed shape can be at least approached by actual atmospheric agencies giving rise to angels. Such a common figure is the cylinder very nearly approached by convective clouds, particularly under situations where convection is not very vigorous or on a large scale; only difference being that instead of having a flat top the clouds have a cap which may be regarded as a section of a sphere. It is intended here to use this combination of cylinder and the spherical section as unit for back-scattering treatment and to discuss the possibility of obtaining eddies of such shape in actual angel producing conditions.

2. Theory

The radar cross-section of a sphere of radius \( a \) for the geometrical case is proportional to \( a^2 \). For a cylinder of radius \( a \) and length \( l \) the corresponding cross-section is proportional to \( a l^2 \). Under identical refractive conditions therefore, the back-scatter from a cylindrical target is likely to be considerably larger than from a spherical one. Thus, while dealing with the problem of radar echoes from refractive gradients, if we can visualize the existence of atmospheric eddies of cylindrical rather than spherical shape, the radar cross-section with a given gradient will be considerably larger. Such a concept would obviate the necessity of unduly large and unrealistic spherical surfaces as required by the near zone theory.

Considering the 'uncondensed cloud' model of angel source mentioned earlier, the total scattering from the 'blob' will consist of two parts: first from the cylindrical portion and second from the spherical cap and the cross-section from the source can be written as (Kerr 1951):

\[
\sigma \approx -\frac{2\pi a^2 l^2}{\lambda} \cos \theta \left[ \frac{\sin(kl\sin\theta)}{kl \sin\theta} \right]^2 + \pi \beta \beta^2
\]

(1)

Where \( h \) is the radius of curvature of the spherical surface of the cap and \( \beta \) is the ratio of the cross-section of the cap to that of a metal sphere with radius \( h \); \( \alpha \) is a constant analogous to the reflectivity term in the plane layer case (Swingle 1953) and \( \theta \) is the angle that the direction of propagation makes with the base of the cylinder. As discussed by Atlas (1960) a part of the sphere surface sufficient to include an odd number of Fresnel zones can give a cross-section even greater than that of the entire sphere. However, here we are concerned with a section of the sphere surface on which the radar waves are incident in such a way that the back-scattering from only that portion in which the beam is almost normally incident will be significant. At near horizontal incidence therefore, scattering from the cylindrical portion will be predominant. With increasing aerial elevation it will rapidly decrease while the contribution from the cap will become increasingly significant and at very high angles the scattering will be almost entirely from the spherical surface. As even for a moderate ratio of \( l \) to \( a \) the cross-section of a cylinder is considerably greater, at horizontal incidence the back-scattered signal will be more pronounced. This may perhaps be one of the reasons that angels on PPI have been observed at much greater ranges.

In actual atmosphere such angel source can normally be expected to occur in any convective or turbulent region. In a region of turbulent convection one can visualize the existence of air 'blobs' consisting of a homogeneous refractive medium embedded in an environment with different refractive properties. In analogy with the usual convective clouds, these may be assumed to have a vertical dimension appreciable larger than the horizontal and thus to have the desired shape. The echoes from a particular region will be a result of the total scattering
from a number of such 'blobs' in the pulse volume. Such conditions may occur in situations like one giving rise to strato-cumulus below subsidence inversions. It may also be realized in the presence of moist layers aloft which under favourable conditions result in alto-cumulus or high strato-cumulus formation.

In many angel studies (Atlas 1959, 1960, Plank 1956, 1959) considerable attention has been given to the bubble theory of convection (Scorer and Ludlam 1953) where convective bubble provides the required large sphere. Such large spheres may be present only in large scale convection often resulting towering cumulus formation. The echoes from such bubbles will exhibit appreciable vertical movement as observed with mantle echoes (Harper et al. 1957, Atlas 1959). In regions of turbulent convection of limited vertical extent, it is probably more realistic to assume eddies of smaller dimensions such as discussed above.

3. Observations

We consider here some angels observed on PPI (Ligda and Bigler 1958, Bigler 1959, Rai 1959, 1960, 1961). The inclination of the beam with the horizontal in these cases is very small so that \( \theta \) in equation (1) is of the order of a few degrees only. Consequently \( \cos \theta \) is very nearly unity and the term in the parenthesis being of the form \( (\sin x)/x \) is also nearly unity. The radar cross-section of the cylindrical eddies in these cases will be approximately \( 2 \pi a^2 / \lambda \). For the thin line echoes at considerable ranges also, this approximation holds good. Only at short ranges \( \theta \) will be appreciable and equation (1) will have to be used.

(i) Angels observed with 'Decca 41' at Bombay—A number of angel observations from Decca 41 radar based at Bombay had been reported and discussed earlier by the author (1959, 1960, 1961). One particular type not presented earlier, which has been observed on some occasions is given in Fig. 1. The echoes observed on 12 April 1960 are
the strongest of this type and occur in association with markedly high temperatures in the lower levels. The echoes have sharp inner and outer edges, generally confined to the SW-NW sector at ranges about 10-20 miles. At closer examination these echoes appear to consist of lines parallel to the edges. With the type of echoes observed on 16 February 1960 only the inner edge is well defined; the outer portions are less marked and appear more as a cluster of distinct dots. The intensity falls off gradually with the range. A noteworthy feature is the wavy structure of the inner edge as in the case of earlier angel observations from the station. In case of the echoes observed on 24 February 1960 the outer edge is sharp while the inner edge is diffuse. The diffuse trailing echoes from the inner edge have some similarity with the streamers in the rear of thin line echoes from a cloudless cold front reported by Ligda and Bigler (1958). No direct evidence of the vertical structure of these echoes is available. However, they are usually obtained with an aerial elevation of 1-2° and can be seen up to 3-4°, although in particular cases (as on 12 April 1960) they persist even up to 5-6° elevation.

The corresponding wind and temperature distribution in the lower levels as obtained from the routine radiosonde observations at Bombay is given in Fig. 2.

As remarked in an earlier communication (Rai 1961) the line type of angel echoes in this region are obtained exclusively during the period February-May in the late afternoons only and low level subsidence inversions are almost a seasonal feature. Let us consider such an inversion at a height $Z$ in a coastal region. During the morning hours when there is no differential heating, the height of the inversion layer is the same over land and water surfaces. In the afternoon when effect of ground heating becomes pronounced, the temperature over land increases to a greater extent with the lapse rate extending to a greater height. Consequently the inversion layer over land will be lifted
up to a greater extent and will be higher than that over the water surface. This would result in a break in the inversion layer as indicated in Fig. 3. The distance from the coast line at which this discontinuity occurs will depend upon the degree of insolation and also on the prevailing winds in the levels concerned. As the inversion layer acts as a lid for the convection which might result due to the insolation, this discontinuous region will be highly turbulent, and may be expected to contain a large number of angel 'blobs' or eddies discussed earlier. On days when the refractive gradient in the stratified layers associated with the inversion is sufficiently high, these eddies will provide the required source for the angels. Some of the angels observed in this region can easily be attributed to back-scatter from the turbulent region. However in the case of the observations presented here, the intensity is too high to be explained by any conceivably large refractive gradient, particularly when one considers the relatively low peak power of the radar utilised.

It appears more probable that the radar waves are predominantly scattered towards the sea and the corrugated sea surface, in turn back-scatters sufficient energy to reach the receiver. This would of course, require that the scatter function of the eddies for a particular direction \( \theta \Delta f/2 \) is a maximum. The parallel striations in the echoes are, therefore, most probably a manifestation of the sea waves in the particular region.

An examination of Fig. 2 shows that low level inversion is present on all these days in the morning hours while in the evening the inversion appears to be wiped off. As it reappears during subsequent hours (as evidenced by the next morning ascent) it may be reasonably assumed that in the afternoon the base of the inversion layer is lifted up and eventually the layer becomes too shallow to be detected by the conventional radiosonde. As there is no direct observation of the vertical extent of these echoes and as the routine radiosonde observations are too inadequate for the purpose of microwave propagation studies, it does not seem worthwhile to attempt any quantitative estimation of the height and extent of the discontinuous region in the inversion layer or of the refractive gradient associated with the eddies.

The sharpness of the edges of these echoes is, in all probability, dependent on the prevailing winds in the source region. Winds with off-shore component would tend to make the inner edge sharper while with on-shore winds, the outer edge will be more pronounced. The sharpness of both the edges might be due to the wind direction being parallel to the coast line.

(ii) Angel bands—Layer type of echoes have been frequently observed at a number of places (Atlas 1959, Bigler 1959) and are usually found to be associated with sandwiched moist layers at the corresponding height. Such a moist layer in which condensation does not result will be an ideal source for the required angel 'blobs'. In these cases, however, the vertical dimensions will
not be very large. But, as discussed earlier, in such cases the predominant scattering will be from the spherical cap and the cylindrical portion of the cloud need not even be considered. With increasing aerial elevation (in RHI or REI scanning) the intensity of the echoes will rapidly increase, partly due to the decrease in range and partly due to the increasing cross-section of the cap particularly because of the focusing effect of the cap becoming more important. In general the intensity of these layer echoes will be a function of the humidity gradient across the layer.

Two examples of an interesting type of angel echoes observed at Delhi with N.M.D 451-A radar* by Rain and Cloud Physics Research Unit, National Physical Laboratory and kindly made available to the author are presented in Fig. 4. The echoes appear in the form of well defined horizontal bands like the melting band echoes and in view of this similarity we term them as angel bands. These bands have been observed with clear sky as well as in cloudy or overcast conditions. When observed under cloudy conditions, visual observations have shown that no cloud has been present at or near the levels corresponding to the bands. The echoes are incoherent in character as seen on the A-scope.

On 2 September 1959 at the time of observation sky was almost clear. 0630 hours observation at Safdarjung, which is about 5 miles to the southeast of the radar station, showed one okta of cumulus while at Palam, roughly 6 miles to the southwest, only two oktas of cirrus was reported. The temperature and dew point profiles as obtained from the routine radiosonde observation at 0530 IST are given in Fig. 5 and clearly indicate the presence of a moist layer at a height of about 1.5 km which is approximately the height of the angel band. A comparison of the two observations on this day suggests

*Set characteristics: $\lambda = 3$ cm, $P_{t} = 2.5 \times 10^{6}$ watts,
$0 = 0 = 1^\circ$, $\tau = 1\mu$ sec, P.R.F. = 300 p.p.s., $P_{r}$ min = 86 dB below one milliwatt
that either the angel layer was not very extensive or the conditions in the layer had been changing rapidly. In the photograph taken at 0620 IST along 112° azimuth, the band can be seen extending up to 15 km while in photograph taken only five minutes later along 002° azimuth, it is extending only upto 10 km.

Observations on 22 September 1959 show two angel bands one at about 2 km and the other at about 3.5 km. The lower one is more pronounced and extends right upto 20 km while the upper one is much weaker extending only upto 10 km. Weather at the time of observation was cloudy, Safdarjung observation at 1130 IST indicating one okta of stratocumulus and the rest altocumulus, the total cloud amount being four oktas. The clouding at the time of observation was comparatively less. The temperature and dew point profiles in respect of 0530 IST radiosonde observations are given in Fig. 5. The presence of two distinct layers at 2 and 3.7 km can be detected although the humidity gradient associated with the lower layer is not so marked. However, in this case the angel observations were made well over five hours after the time of the sounding and during this period considerable changes might have occurred to affect the scattering character of the layer.

It may be mentioned here that these angel observations have been obtained during routine operation of radar for cloud and precipitation studies. The observations are, therefore, not available in sequence and the photographed angel echoes do not necessarily have the corresponding intensity observations. Further the time of observation rarely falls close to the time of radiosonde ascents. As conditions favourable for angel echoes may change considerably during a few hours period, it is rather difficult to make a quantitative comparison except in a few cases. However, even with this limited number of observations the association of these angel bands with moist layers aloft is unmistakable.

4. Conclusion

Considering some angels observed with horizontal scanning and the layer type of angels observed with vertical scanning we have seen that it is not possible to treat these as near zone cases. At the same time the commonly assumed spherical model of the atmospheric bubble with small radius of curvature is incapable of giving the required radar cross-section with a reasonably large refractive gradient. A possible alternative is to assume a cylindrical model of the eddies. A source closely approaching such a shape may be the moist air parcels in the form of the usual convective clouds with spherical cap. The cross-section of such a source with a given refractive gradient, will be considerably larger than that of a spherical source. At horizontal incidence, the scattering from the cylindrical portion will be the predominant one while at high angles of aerial elevation, it will be mostly from the spherical cap. Such angel source may be provided by any region of turbulent convection. The angels observed along west coast near Bombay, as well as the layer type of echoes observed at Delhi can be explained with this model. Some remarkably intense echoes observed over the coastal region near Bombay, however, appear to be caused by return from the sea surface towards which the energy is strongly scattered by the turbulent discontinuous region in the
inversion layer. It appears therefore, that most of the angel echoes are directly or indirectly caused by regions of turbulent convection. In the author's opinion, a plane layer with any reasonably large refractive gradient, is not capable of giving rise to these echoes except at normal incidence and at comparatively short ranges.

REFERENCES

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