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L E T T E R S

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A CLIMATIC PREDICTABILITY INDEX FOR SOUTH WEST MONSOON SEASON IN DIFFERENT DISTRICTS OF WEST BENGAL WITH APPLICATION OF FRACTAL DIMENSION ANALYSIS

1. Investigation of the relationship among climatic variables namely, temperature, vapour pressure and rainfall significantly play a predominant role in building model and prediction through modelling in the Himalayan and dooars region along with Gangetic plains but indicates limitations of the efficiency of the model due to complicated geographical topography (Pant *et al.*, 2018: Singh *et al*., 2016). The statistical variations among climatic variables limit one to point out the relationships among those and are lacking over some of the regions.

Temporal dynamics can be formulated by considering the observations of different climate variables as a multi-dimensions arrays and combining fractal-based dynamics with clustering (Mingkai *et al*., 2016). Number of methods has been applied for analyzing the non-linear dynamics in the time series by noble researches (Eckmann and Ruelle, 1985; Tong, 1993; Diks, 1999). The predictability of climate models rests on the combination of dynamic and thermodynamic processes that results in non-linear responses in the atmosphere; thus for a proper identification, classification and mapping climate variations, long-term systemic observational data sets are required from a network of stations or districts in connection with the statistical technique.

It is well known that rainfall events or variables are highly important not only for scientific purpose but also for all possible environmental and atmospheric purposes. However, with research field changed drastically with the contribution of Hurst and Mandelbrot, as well as many studies performed all over the world, the research leads to the relation between the variable and fractal behaviour (Turcotte, 1997; Peters, 1996).

Fractal theory has been widely applied on diverse data sets in geophysics as well as climatological fields (Mandelbrot and Wallis, 1969; Rehman, 2009) to identify the pattern in time series data sets for describing irregular and complex behaviours of dynamical systems (Men *et al*., 2004). He and Gautam, 2016 analyzed the spatial-temporal variation in rainfall for flood seasons during 1958-2013 utilizing Hurst exponent in China and concluded that the rainfall would persist in future and have implication for the ecological restoration and farming operations.

For this reason, we have chosen to work with fractals instead of methods involving probability, given that dynamic systems display in nature in self similarity and space-time fluctuations on their behaviour on all scales, in indicating correlation on large scale.

The state of West Bengal is located at the east of Himalayan region and the Gangetic plains with the border with Bangladesh. Thus, the state has high impact on the climatic related risks on economy and agriculture along with the dense population. It is an important issue to find out the climate of each of the districts of the state for climate related issues.

It is necessary to note that fractal dimension analysis is used as a necessary tool for few decades for the geophysical and climatological time series such as temperature, pressure as evidenced in the novel works of several scientists (Hurst, 1965; Mandelbrot and Wallis, 1969; Fluegeman and Snow, 1989; Hsui *et al*., 1993; Turcotte, 1997; Rangarajan and Sant, 2004). These analyses have been concentrated on computation of fractal dimensions for individual time series. In the present paper, we attempt to link these dimensions to dynamics of climate as the underlying processes are dynamically linked together. Firstly, for individual series in a district, the Hurst exponent of the individual is extracted (Mandelbert and Wallis, 1969). Thereafter, th fractal dimension (Vose *et al*., 1992) is extracted for each of the climatological series. In the next stage, the Climatic indices (Rangarajan and Sant, 2004) are computed. We attempt to analyse the character of the indices and study the effect of the indices for the districts of the state of West Bengal. The index is shown to be useful in studying the interplay between various climatic components.

The organisation of the paper is as follows. Firstly, we compute the Hurst exponent of the climatological series of temperature, vapour pressure and rainfall of South West Monsoon (SWM) season for each of the 18

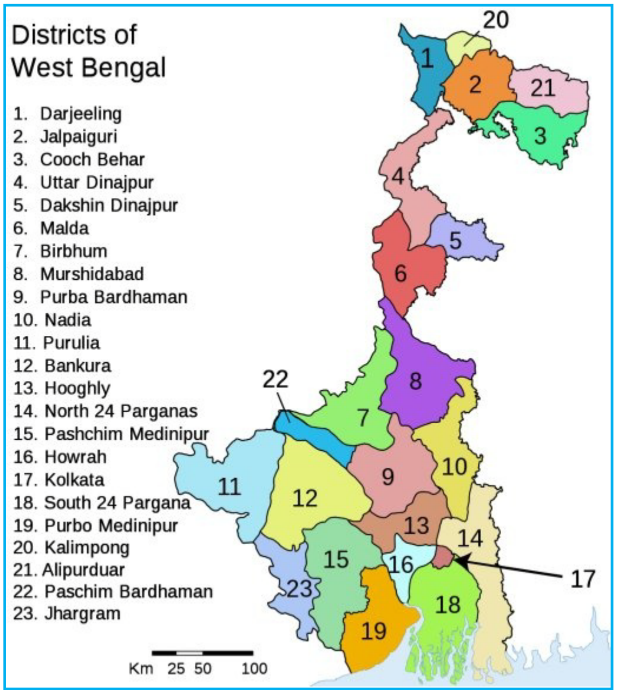

Fig. 1. District wise map of West Bengal state

districts of the state of West Bengal, India. A district wise map is shown in Fig. 1.

Thereafter, for each of the districts time series of temperature, vapour pressure and rainfall, Fractal dimension is computed utilizing the linkage formula between Hurst exponential and fractal dimension as explained in the next section.

After obtaining the fractal dimension, the climate predictability indices of each of the series under consideration is obtained.

2.1. *Data* - The data set of temperature, vapour pressure and rainfall in South West Monsoon (SWM) season of 18 districts for 100 years (1901-2000) are extracted from the website 'www.indiawaterpool.in'. The temperature data (in °C), collected as monthly average for each year, are summed up for the months June to September to compute year wise SWM temperature data. In the same way, the vapour pressure (in "Hg) is assembled to extract the year wise SWM vapour pressure data. However, rainfall data (in mm) extracted as year wise monthly total and are assembled for SWM months to set up year wise SWM rainfall data.

2.2. *Hurst Rescaled Range Analysis* - An approach to the correlation quantification in time series was developed by Hurst (1965) based on empirically introduced concept of R/S. Considering a time series, the summation of time series relative to their average value is

$$y_n = \sum_{i=1}^{n} \left(y_i - \bar{y}_N\right) = \left(\sum_{i=1}^{n} y_i\right) - n\bar{y}_n \tag{1}$$

where the range is defined by

$$R_N = (y_n)_{\max} - (y_n)_{\min} \tag{2}$$

$$\text{with } S_N = \sigma_N \tag{3}$$

where $\bar{y}_N$ and σ_N are the mean and standard deviations of all the N values in the time series y_n. Proceeding this way, from the previous equations, a value (R_N / S_N) is obtained for the time series y_n. We substitute τ by N in equations (1)-(3). The Hurst exponent (H) is obtained as

$$\left(\frac{R_\tau}{s_\tau}\right)_{av} = \left(\frac{\tau}{2}\right)^H \tag{4}$$

The rescaled range (R/S) (w) is defined as:

$$(R/S)(w) = \left\langle \frac{R(w)}{S(w)} \right\rangle \tag{5}$$

where w is the window width and the symbol < > represent the average values of a number of values of $R(w)$. Based on the self-affinity, it can be expected that:

$$(R/S)(w) = w^H \tag{6}$$

In reality, for a determined value w, a time series is sub divided by a number of intervals of width w, then $R(w)$ and $S(w)$ are calculated for each one and $(R/S)(w)$ as the average ratio $R(w)/S(w)$.

The procedure mentioned above is repeated for determined number of window widths and the logarithm of $(R/S)(w)$ are plotted against the logarithm of w. If the set has self-affinity, then plot will follow a straight line whose slope is equal to the Hurst Exponent H.

H describes the correlation between the past and future in the time series. For independent random process with finite variances, the value of H is 0.5.

When $H > 0.5$, the time series is persistent meaning that an increasing trend in the past is indicative of an increasing trend in the future. Conversely, as the reverse rule, a decreasing trend in the past signifies a persistent decrease in the future.

TABLE 1

Values of H, D and PI for districts spread over the state of West Bengal in SWM season

Districts	H_T (Temp)	$D_T = 2 - H_T$	$PI_T = 2\lvert D_T - 1.5\rvert$	H_P (Vapour Pressure)	$D_P = 2 - H_P$	$PI_P = 2\lvert D_P - 1.5\rvert$	H_R (Rainfall)	$D_R = 2 - H_R$	$PI_R = 2\lvert D_R - 1.5\rvert$	$PI_r = (PI_T, PI_P, PI_R)$
Purulia	0.3780	1.6220	0.2440	0.3860	1.6140	0.2280	0.3988	1.6012	0.2024	(0.3544, 0.2280, 0.2024)
Mursidabad	0.3685	1.6315	0.2630	0.6067	1.3933	0.2134	0.6772	1.3288	0.3544	(0.2630, 0.3124, 0.3424)
Maldah	0.5486	1.4514	0.0972	0.6341	1.3659	0.2682	0.8385	1.1615	0.6770	(0.0972, 0.2682, 0.6770)
Kolkata	0.3427	1.6573	0.3146	0.5848	1.4152	0.1696	0.3403	1.6597	0.3194	(0.3146, 0.1696, 0.3944)
Bankura	0.3659	1.5341	0.2682	0.4659	1.5341	0.0662	0.3177	1.6823	0.3646	(0.3190, 0.0682, 0.3646)
Uttar Dinajpur	0.4497	1.5503	0.1006	0.4497	1.5503	0.1006	0.8310	1.1690	0.6620	(0.1006, 0.300, 0.6620)
South 24 pgs.	0.6662	1.3338	0.3324	0.5958	1.4052	0.1896	0.4876	1.5124	0.0248	(0.3324, 0.1912, 0.0678)
Medinipur	0.6850	1.3150	0.3700	0.4160	1.5840	0.1680	0.3104	1.6896	0.3792	(0.2630, 0.1680, 0.3446)
Coochbihar	0.4831	1.5169	0.0338	0.6983	1.3017	0.3966	0.3177	1.6823	0.2642	(0.0338, 0.3996, 0.2602)
Jalpaiguri	0.5542	1.4458	0.1084	0.7858	1.2142	0.5716	0.6718	1.3282	0.3436	(0.1084, 0.5716, 0.3436)
Howrah	0.3776	1.6224	0.2448	0.5617	1.4383	0.1234	0.4199	1.5801	0.1602	(0.2446, 0.1234, 0.1602)
Darjeeling	0.6650	1.3350	0.3300	0.8990	1.1010	0.7980	0.8161	1.1839	0.6322	(0.3300, 0.7980, 0.632)
Dakhshin Dinajpur	0.4143	1.5857	0.1714	0.5477	1.4523	0.0954	0.8280	1.1720	0.6560	(0.1714, 0.1154, 0.6560)
Birbhum	0.3850	1.6150	0.2300	0.5248	1.4052	0.1896	0.7818	1.2182	0.5636	(0.2300, 0.1896, 0.5638)
Bardhaman	0.3974	1.6026	0.2052	0.6124	1.3876	0.2248	0.1618	1.8382	0.6764	(0.2052, 0.2248, 0.4750)
North 24 pgs.	0.4780	1.5220	0.0440	0.5256	1.4744	0.0512	0.6775	1.3225	0.3550	(0.1926, 0.1000, 0.2375)
Hoogli	0.3879	1.6121	0.2242	0.4781	1.5219	0.0438	0.4199	1.5801	0.1602	(0.2242, 0.0438, 0.1602)
Nadia	0.4781	1.5219	0.0109	0.5721	1.4280	0.1440	0.6773	1.3227	0.3546	(0.0109, 0.1440, 0.3546)

On the other hand, when $H < 0.5$, the time series is anti-persistent meaning that an increasing trend in the past implies decreasing trend in the future and *vice-versa*.

Lastly, if H is almost equal to 0.5, it indicates that the time series concerned is random.

2.3. *Computation of Fractal Dimension* - The Hurst exponent H is related to the fractal dimension D of the time series curve by the Hurst exponent-fractal dimension formula $D = 2 - H$ (Voss, 1985; Vose *et al.*, 1992).

If the fractal dimension D for the time series is 1.5, there is no correlation between amplitude changes corresponding to two successive time intervals. Therefore, no trend in amplitude can be discerned from the time series and hence the process is unpredictable (Mandelbrot and Wallis, 1968).

However, as the fractal dimension decreases to 1, the process becomes more and more predictable as it exhibits ‘‘persistence’’. That is, the future trend is more and more likely to follow an established trend (Hsui *et al*., 1993).

As the fractal dimension increases from 1.5 to 2, the process under consideration exhibits behaviour of “anti-persistence”. That is, a decrease in the amplitude of the series is more likely to lead to an increase in the future and vice-versa. Naturally, the predictability again increases.

However, it may be mentioned that almost all geophysical and climatological time records analyzed (Mandelbrot and Wallis, 1969; Fluegeman and Snow, 1989; Hsui *et al*., 1993) exhibits persistent behaviour.

2.4. *Formulation of index* - We obtain the fractal dimension of the time series corresponding to temperature, vapour pressure and rainfall in SWM season for a given location using R/S analysis. First we obtain H and then equation ($D = 2 - H$) (Voss, 1985; Vose *et al.*, 1992) is utilized to extract fractal dimension. The fractal dimensions denoted by D_T, D_P and D_R correspond to temperature, vapour pressure and precipitation respectively.

Particularly, indices P_T, P_P and P_R for temperature, vapour pressure and rainfall are defined as follows (Hurst *et al.*, 1993; Fluegeman and Snow, 1989; Turcotte, 1997).

$$P_T = 2|D_T - 1.5|;\ P_P = 2|D_P - 1.5|;\ P_R = 2|D_R - 1.5|;$$

where $|D|$ denotes the absolute value of the number D. The absolute value is used since predictability increases in both the following cases-(a) when the fractal dimension becomes less than 1.5 and (b) when it becomes greater than 1.5. In the case (a), we have persistence (correlation) behaviour and in case (b), we have anti-persistence (anti-correlation) behaviour. In either case, the process retains same predictability with $D = 1.3$ and $D = 1.7$.

It is noted that climate predictability index PI_C is defined as collection of 3 indices, namely, (Rangarajan and Sant, 2004):

$$PI_C = (P_T, P_P, P_R)$$

If one of these indices is close to zero, then the corresponding process approximates the usual Brownian motion and it is therefore unpredictable.

If one of the indices is close to 1, the process is very much predictable. We would note that P_R value is not related to amount of rainfall but to how precipitation value changes from year to year. It is the predictability index. The same logic holds good for temperature and vapour pressure also. In this paper, we are interested in the interrelationships between the two climatic components from a view point of fractal dimension. Then, it is useful to have all three of sub-indices represented as a single index PI_C. It is easier to see how the 3 sub-indices changes from district to district. Also, introducing predictability indices instead of fractal dimension, we focus how predictable the process is, specially the precipitation. All the sub-indices (*viz*., P_T, P_P, P_R, temperature, vapour pressure and rainfall in SWM season) for all the 18 districts are presented in Table 1.

Sometimes, it may be argued that three sub-indices may be presented into a single number using appropriate norm. This may not be quite all right as the processes are independent of others. For that, we formulate PI_C as a collection of three climatological parameters of the particular location that has vital role in determining the weather of the region. One important factor which has not explicitly included in making up PI_C is the geographical parameters in the location, but the fact is that the above sub-indices include the effect of geographical parameters implicitly.

3. *Application*- We arrive at the actual calculation of PI_C, presented in Table 1. As mentioned, we concentrated on the temperature, vapour pressure and rainfall series of SWM season of 18 districts spread over the state of West Bengal. From a climatic point of view, we understand that summer heat leads to low pressure over the Tibetan plateau, northern India including state of West Bengal, induces a strong monsoonal flow over the region from Bay of Bengal in SWM monsoon during the months June to September.

We first extract separately temperature, vapour pressure and rainfall series of SWM season. Fractal analysis is performed for each of the categories as mentioned in the section 2.2-2.4. Firstly, R/S is computed as mentioned in section 2.2. From the R/S analysis, Fractal Dimension and also the climatic index PI_C is computed as mentioned in the last sections 2.3 and 2.4. We restrict our attention mainly to the districts where any of the entries of PI_C, that is, P_T, P_P or P_R are greater than or equal to 0.4 and assign the district *strongly predictable* (denoted by S) with respect to that entry. For example, if P_R is ≥ 0.4 for Darjeeling district, then we assign Darjeeling as strongly predictable for rainfall.

However, if the entries are between 0.1 and 0.4, then the concerned district would be assigned as weakly predictable (denoted by W) with respect to that entry.

Lastly, if the entries are less than 0.1, the district would be considered as *unpredictable* with respect to that entry (denoted by U).

3.1. *Temperature* **-** Investigation of Table 1 indicates that temperature index is mainly *weakly* predictable ($0.1 < PI_T < 0.4$) (Rangarajan and Sant, 2004) for all the districts under investigation. Except a few cases, it is weakly predictable irrespective of geographical position of the districts starting from hilly areas like Darjeeling, dooars area like Jalpaiguri to semi-arid areas like Mursidabad, Bankura, Uttar and Dakhshin Dinajpur and dry zones of Birbhum, Maldah, Purulia and districts over gangetic plains like Bardhaman, Hoogli, Howrah, South 24 parganas and Medinipur. It may be understood that the nature of temperature distribution is such that they are not completely random but keeps possibility of some kind of model with less efficiency for all the districts. The model thus generated would not be very much efficient and accurate.

However, the temperature index is unpredictable ($P_T \leq 0.1$) for the districts Coochbihar, Maldah, North 24 parganas and modelling exercise would not be fruitful.

3.2. *Vapour pressure* - Table 1 indicates that Jalpaiguri and Darjeeling districts possess remarkably *high* predictability of Vapour pressure ($PI_P \geq 0.4$). It indicates strong persistence or anti-persistence in those series. As a result, the increasing /decreasing tendency of the series, it is followed by increasing /decreasing tendency or decreasing/increasing tendency. These tendencies can be modelled with a suitable empirical formula.

However, the districts Kolkata, Medinipur, Howrah, Nadia, Bardhaman and South 24 parganas are located at the plain and Purulia, Mursidabad, Malda, Coochbihar, Birbhum located at the semi-arid region possess *weak* predictability. A modelling exercise may not be very efficient for the districts.

The unpredictable districts ($PI_P \leq 0.1$) are Hoogli, Bankura, Dakhsin Dinajpur and North 24 parganas. Modelling exercise may not be fruitful for those districts.

3.3. *Rainfall* **-** It is observed from Table 1 that rainfall in SWM season in the districts of Maldah, Bardhaman, Uttar and Dakhshin Dinajpur, Birbhum and Darjeeling are strongly predictable ($PI_R \geq 0.4$). The districts Maldah, Uttar Dinajpur and Dakhshin Dinajpur, Birbhum are situated at the semi arid region of the state of West Bengal. Darjeeling district is located in the upper hilly portion of the state and Bardhaman is located at the Gangetic plains of the state. Accordingly, a suitable model may be constructed with selected parameters. Moreover,

TABLE 2

Quality of Predictability index of Darjeeling district

Predictability indices	P_T	P_P	P_R
Nature of prediction	W	S	S

W: ($0.1 < PI < 0.4$), S: ($PI \geq 0.4$)

TABLE 3

Quality of Predictability index of Jalpaiguri district

Predictability indices	P_T	P_P	P_R
Nature of prediction	W	S	W

W: ($0.1 < PI < 0.4$), S: ($PI \geq 0.4$)

SWM rainfall is *weakly* predictable ($0.1 < PI_R < 0.4$) for the districts Purulia, Mursidabad, Kolkata, Howrah, Hoogli, Coochbihar, Jalpaiguri, Nadia, Bankura, North 24 parganas, of which Purulia, Mursidabad, Bankura are semi-drought prone regions and Kolkata, Howrah, Hoogli, Nadia and North 24 parganas are located at the Gangetic plains of the state along with Coochbihar and Jalpaiguri located at the plains in the lap of Himalaya. Only South 24 parganas in the Gangetic plain does not show *any* predictability at all.

4. *Identification of patterns of climatic index of different districts of West Bengal* - Examination of climatic indices (P_T, P_P, P_R) of districts leads to identify some interesting features. ./As mentioned earlier, an index, say P_T would be *strongly* predictable if its magnitude is greater than or equal to 0.4, *i.e.*, $P_T \geq 0.4$ denoted by ‘S’, weakly predictable, if $0.1 < P_T < 0.4$, denoted by ‘W’ and unpredictable, if $P_T \leq 0.1$, denoted by ‘U’ (Rangarajan and Sant, 2004). The *seven* identifiable patterns emerged in the following way.

4.1. *Uniqueness of pattern of climatic predictability of Darjeeling district* **-** The Table 2 extracted from Table 1 indicates that for Darjeeling district temperature predictability (P_T) is weak; however, vapour pressure (P_P) and rainfall predictability (P_R) are strong. This indicates that for temperature, the previous year’s data is not likely to interpret the future temperature data in the correct way.

The situation is quite different for vapour pressure and rainfall indices as P_P and P_R are very strong. The previous increasing/decreasing pattern of vapour pressure and SWM rainfall are likely to indicate correct prediction pattern of increasing/decreasing or decreasing/increasing pattern in future.

TABLE 4

Quality of Predictability index of South 24 Parganas district

Predictability indices	P_T	P_P	P_R
Nature of prediction	W	W	U

W: (0.1 < PI < 0.4), U :(PI ≤ 0.1)

TABLE 5

Quality of Predictability index of Purulia, Murshidabad, Midnapur, Kolkata, Howrah and North 24 Parganas districts

Predictability indices	P_T	P_P	P_R
Nature of prediction	W	W	W

W: (0.1 < PI < 0.4)

TABLE 6

Quality of Predictability index of Maldah, Birbhum, Bardhaman, Uttar Dinajpur and Dakhshin Dinajpur districts

Predictability indices	P_T	P_P	P_R
Nature of prediction	W	W	S

W: (0.1 < PI < 0.4), S: (PI ≥ 0.4)

TABLE 7

Quality of Predictability index of Hoogli and Bankura districts

Predictability indices	P_T	P_P	P_R
Nature of prediction	W	U	W

W: (0.1 < PI < 0.4), U :(PI ≤ 0.1)

The district is located at the hilly area of Himalayan range.

4.2. *Uniqueness of pattern of climatic predictability in Jalpaiguri district* **-** The pattern of climatic index of Jalpaiguri is displayed in the following Table 3. The table indicates that vapour pressure of the district Jalpaiguri is *strongly* predictable indicating that previous/increasing/ decreasing pattern likely to indicate correct prediction pattern of increasing/decreasing or decreasing/increasing pattern in future; however, temperature and rainfall are *weakly* predictable indicating that for temperature and rainfall, the previous years' data is not likely to interpret the future temperature and rainfall patterns. The district is located at the dooars area of sub Himalayan region.

4.3. *Uniqueness of pattern of climatic predictability in South 24 parganas district* **-** The pattern of climatic index of South 24 parganas is displayed in Table 4. The table displays that both temperature and vapour pressure are weakly predictable indicating possibility of model with less effiency whereas the rainfall is unpredictable. The district is located at the Gangetic plain of the state.

4.4. *Similarity of pattern of climatic predictability of districts Purulia, Murshidabad, Kolkata, Midnapur, Howrah and North 24 parganas districts* **-** The Table 5 indicates that all the predictability indices such as temperature, vapour pressure and rainfall are weakly predictable for the districts Purulia, Kolkata, Murshidabad, Midnapur, Howrah and North 24 parganas. All the districts are located in the Gangetic plains of the state except the districts Purulia and Mursidabad which are located at the dry and semi-arid zones of the state. The previous pattern of increasing/decreasing pattern of temperature, vapour pressure and SWM rainfall are likely to indicate inaccurate prediction pattern of increasing/decreasing or decreasing/increasing pattern in future.

4.5. *Similarity of pattern of climatic predictability of districts Maldah, Birbhum, Bardhaman, Uttar Dinajpur and Dakhshin Dinajpur* - For the districts of Maldah, Birbhum, Bardhaman, Uttar Dinajpur and Dakhsin Dinajpur in terms of predictability form a common Table 6.

The Table indicates that the predictability indices such as temperature and vapour pressure are weak for the districts Maldah, Birbhum, Bardhaman, Uttar Dinajpur and Dakhshin Dinajpur. However, for these districts, rainfall predictability indices are sufficiently strong so that a proper model to be built up considering the persistency or anti-persistency of the rainfall series where SWM rainfall occur in abundance. The rainfall mostly boosts agriculture and economy in the state.

Of the districts, Bardhaman is located at the Gangetic plains of the state and other districts are at the semi-arid region of the state.

4.6. *Similarity of pattern of climatic predictability of Hoogli and Bankura district* **-** The predictability of the Hoogli and Bankura districts are similar in terms of predictability and are presented in Table 7.

It is observed in the Table that temperature and rainfall are *weakly* predictable*;* however, vapour pressure is totally unpredictable in those districts. Hoogli is located

TABLE 8

Quality of Predictability index of Coochbihar and Nadia districts

Predictability indices	P_T	P_P	P_R
Nature of prediction	U	W	W

W: (0.1 < PI < 0.4), U :(PI ≤ 0.1)

in the Gangetic plain and Bankura is located at the semi-arid region of the state.

4.7. *Similarity of pattern of climatic predictability of Coochbihar and Nadia district* - Table 8 indicated that temperature for the districts of Coochbihar and Nadia is *unpredictable* whereas, vapour pressure and rainfall are very weakly predictable. The contrast of predictability is that Coochbihar is located in the dooars region but Nadia is located in the Gangetic plains of the state.

5. *Conclusion* **-** We now proceed to the actual and realistic conclusion for the construction of climatic index $PI_{C.}$ The three constituents of the index, namely, P_T, P_P and P_R of SWM season are the basic components of a climate of a district. The high or low values of a component indentify the predictability pattern of the component limited to the district. The strong predictability of a component (say, $P_T \geq 0.4$) at a district indicates only strong matching of the model with the actual data set of the district. These points out that particular component can be modelled suitably with a modelling exercise as the pattern of correlation (persistence or anti-persistence) of the model matches with data set to a high extent. A multiple regression or ARIMA model would be suitable in that case.

However, if predictability index is sufficiently low (say, $0.1 < P_T < 0.4$), then the pattern of correlation proceed towards random process and consequently construction of model become more and more complicated, if at all it may be formulated.

The contribution of climatic index highlights one more interesting feature. The effort of simple regression modelling exercise may be of no use if climatic index is *weak* (say, $0.1 < P_T < 0.4$) or *do not exist* at all (say, $P_T \leq 0.1$) pointing out the presence of a random noise (Brownian) in the data. Then, we are to approach for other complicated methods like Principle Component Analysis, Intrinsic Mode Function and other methods for the modelling purpose (Iyenger and Raghukant, 2005; Basak, 2014). Thus, before proceeding into modelling exercise, one has to check the nature of the climatic index for predictability. The present paper points out the interesting feature.

6. *Discussion* - In the current paper, we have taken a step forward in quantification of climatic uncertainties by proposing climatic predictability index. These indices so developed are computed through a fractal dimension analysis of the time series of three major components of climate - temperature, vapour pressure and rainfall in SWM period. These indices provide additional information of how predictable the climate is for a given district. It is particularly useful when a wet/dry season pattern caused by monsoon dominates climate in sub-tropical Asia including the state of West Bengal as a result of planetary atmospheric feature; in general, we indicated that predictability indices change quite significantly as an effect of climate dynamics from district to district. Thus, one has to be very careful in handling year wise data. Also, since predictability indices give a single dimension less number, it can roughly quantify the interplay between temperature, vapour pressure and rainfall in SWM season. The Table 1 explicitly demonstrates how temperature or vapour pressure affects the predictability of rainfall.

PI_C can be useful when developing climate models of a region. In climatic prediction models, one looks for trends in the time series of climatic variables and correlation between those can help specify the model. In that case, one should avoid districts that have a low PI_C components (P_T, P_P, P_R) since data from such a district would contain random amplitude variation, perhaps, caused by local conditions specific to that district. Such anomalous districts distract the entire model.

The shift in emphasis from fractal dimension to predictability may itself be useful as the later concept is more intuitive. Instead of working out in terms of fractal dimension and then making the association with its implications for the time series, one has to straight way switch directly to concept of predictability. Even though we have used predictability in the concept of climate, the basic concept used would have applicability in other fields also.

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