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The gradient of Radio Refractive Index over India

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ABSTRACT. Based on the data of 12 radiosonde stations in India during the year 1965, the exponential and straight line expressions between the radio refractive index near the ground surface and its simple difference from that at 1 km above the ground surface $\triangle N$, have been derived by the least squares method for the four representative months recommended by C.C.I.R. and the values of $\triangle N$ obtained from these equations were subjected to chi-square test of significance.

1. Introduction

The radio refractive index, N_s, near the ground surface is used in many problems of U.H.F. and microwave communications. The main advantage in favour of the use of N_s lies in the fact that the surface observatories are many and the routine meteorological observations of atmospheric pressure, dry bulb and dew point temperatures for computing N_s are taken several times a day, thus facilitating the study of diurnal variation of basic transmission loss. However, experimental investigations have shown that at some places, the observed transmission loss is a function of $\triangle N$, the simple difference between the radio refractive index at 1 km and that near the ground surface. In order to predict $\triangle N$ from largely available data of N_s, statistical methods are employed provided there is a high degree of correlation between $\triangle N$ and N_s.

On the basis of the results available from several western countries, Bean (1962) has concluded that "the observational evidence corroborates the assumption of an average exponential decrease of N with height; the general application of the exponential model must await considerable work on the part of radio climatologist". The mean radio refractive index profiles for tropical maritime and monsoon air masses of India given by Gerson (1948) also showed an exponential height distribution. But Rao and Rao (1966) recently reported a correlation coefficient of $0 \cdot 29$ between $\log \triangle N$ and N_s based only on six radiosonde stations in India thus inhibiting the use of exponential model. This attracted the attention of the authors to undertake the present investigation.

In this paper, the authors have derived the exponential as well as straight line least squares fit between $\triangle N$ and N_s after studying the seasonal variation of the correlation coefficients between $\triangle N$ and N_s. The computed values of $\triangle N$ with the help of these equations were subjected to chi-square test of significance.

2. Modified Radio Refractive Index

The modified radio refractive index, N_s, near the ground surface is expressed by the well-known relation —

$$N_s = \frac{77 \cdot 6}{T} \left(P + \frac{4810\, e}{T} \right) \tag{1}$$

where P is the atmospheric pressure in mb, T is the temperature in degrees absolute, e is the partial vapour pressure in mb of the water vapour present in the air.

The modified radio refractive index, N_{900} at 900-mb level ($\approx$1 km) is given by—

$$N_{900} = \frac{77 \cdot 6}{T} \left(900 + \frac{4810\, e}{T} \right) \tag{2}$$

The absolute value of $\triangle N$ is obtained by the simple difference of N_{900} from N_s.

3. Data

Mean monthly values of pressure, temperature and dew point based on the radiosonde data of 12 stations for the year 1965, for the months recommended by C.C.I.R., *viz.*, February, May, August and November were used in Eqns. (1) and (2) to calculate N_s and N_{900}. The number of radiosonde observations at Minicoy in each month were generally too meagre to be included. Bangalore and Srinagar data were also excluded in view of their orographic and peculiar climatic influence on the modified radio refractive index. The results derived in this paper, therefore, may not hold good for mountainous regions (>3 km) over the country. While deriving the least squares determination between $\triangle N$ and N_s over United States, the data over southern California was excluded in a similar manner since the departure of data points plus the unique nature of the southern California summer climate were taken as sufficient justification for ignoring the points (Bean and Dutton 1966).

TABLE 1

Seasonal variation of N_s, $\triangle N$, $\triangle N$ exp.* and $\triangle N$ St. line† over India

Station	February (N units)				May (N units)				August (N units)				November (N units)			
	N_s	$\triangle N$	$\triangle N$ exp.	$\triangle N$ St. line	N_s	$\triangle N$	$\triangle N$ exp.	$\triangle N$ St. line	N_s	$\triangle N$	$\triangle N$ exp.	$\triangle N$ St. line	N_s	$\triangle N$	$\triangle N$ exp.	$\triangle N$ St. line
Ahmedabad	310	43	37	38	319	45	28	36	376	54	52	53	331	53	49	47
Allahabad	314	43	39	40	309	38	25	31	379	55	54	54	330	52	49	47
Bombay	334	52	49	51	376	80	64	65	383	56	55	56	337	50	51	50
Calcutta	336	53	50	52	377	67	65	65	392	54	59	59	354	63	58	57
Gauhati	331	44	47	49	337	27	37	45	388	48	57	58	352	56	57	56
Jodhpur**	300	31	34	32	318	42	28	36	373	54	51	52	—	—	—	—
Madras	363	70	67	66	375	75	63	64	385	66	56	56	375	70	67	66
Nagpur	306	29	36	35	298	5	21	25	372	44	51	51	316	28	44	41
New Delhi	312	39	38	39	314	45	26	33	378	55	53	54	322	47	46	44
Port Blair	376	72	77	73	384	55	72	69	379	53	54	54	375	60	67	66
Trivandrum	359	67	64	64	389	70	77	72	388	65	57	58	357	57	59	58
Visakhapatnam	365	62	68	67	385	62	73	70	379	53	54	54	374	60	67	65

** Jodhpur observations not available for November 1965

* $\triangle N$ exp. — Value of $\triangle N$ computed from exponential model

† $\triangle N$ St. line — Value of $\triangle N$ computed from straight line expression

The values of $\triangle N$ and N_s calculated from equations (1) and (2) are given in Table 1. The charts showing the spatial distribution of $\triangle N$ as an aid in estimating basic transmission loss and the homoclimes of radio climate of $\triangle N$ have been reported elsewhere (Srivastava (*—see ref.*)

4. Results and Discussion

4.1. *Correlation coefficients* -- The correlation coefficients given in Table 2 show that the exponential as well as straight line least squares fit between N_s and $\triangle N$ appear to be suitable over India except during August when a correlation coefficient of the order of 0·40 was obtained. However, the correlation coefficient for straight line fit is of slightly higher degree as compared to the exponential model.

4.2. *Exponential model* — The exponential model which has received wide application in radio propagation problems is expressed as —

$$\triangle N = A \exp . e^{B.N_s} \qquad (3)$$

where A and B are constants.

The least squares determination is facilitated by converting Eq. (3) in the form—

$$\log_{10} \triangle N = B.\, N_s \times \log_{10} e + \log_{10} A \qquad (4)$$

or expressing the logarithm of $\triangle N$ as a linear function of N_s. The values of A and B which vary with climate were determined by least squares method from 12 radiosonde stations in India for each of the four representative months separately. The following expressions for the corresponding months were obtained —

$$\left.\begin{array}{ll} \triangle N = 1\cdot263\ e^{\,0.010938\ N_s} & \text{Feb} \\ \log \triangle N = 0.006192\, N_s\ -0\cdot52195 & \text{May} \\ \triangle N = 3\cdot267\ e^{\,\cdot007381\ N_s} & \text{Aug} \\ \triangle N = 4\cdot410\ e^{\,\cdot007258\ N_s} & \text{Nov} \end{array}\right\} (5)$$

TABLE 2

Correlation coefficients and χ^2 values

Month	Correlation co-efficients		χ^2 value from	
	$\log \triangle N . N_s$	$\triangle N . N_s$	Exp. fit	St. line fit
Feb	0·93	0·97	4·5644	3·6494
May	0·79	0·83	64·2738	41·1193
Aug	0·42	0·43	6·0024	3·9853
Nov	0·66	0·83	3·5508	3·7948

Note — Values of χ^2 calculated from $\triangle N$ correct to first place of decimal

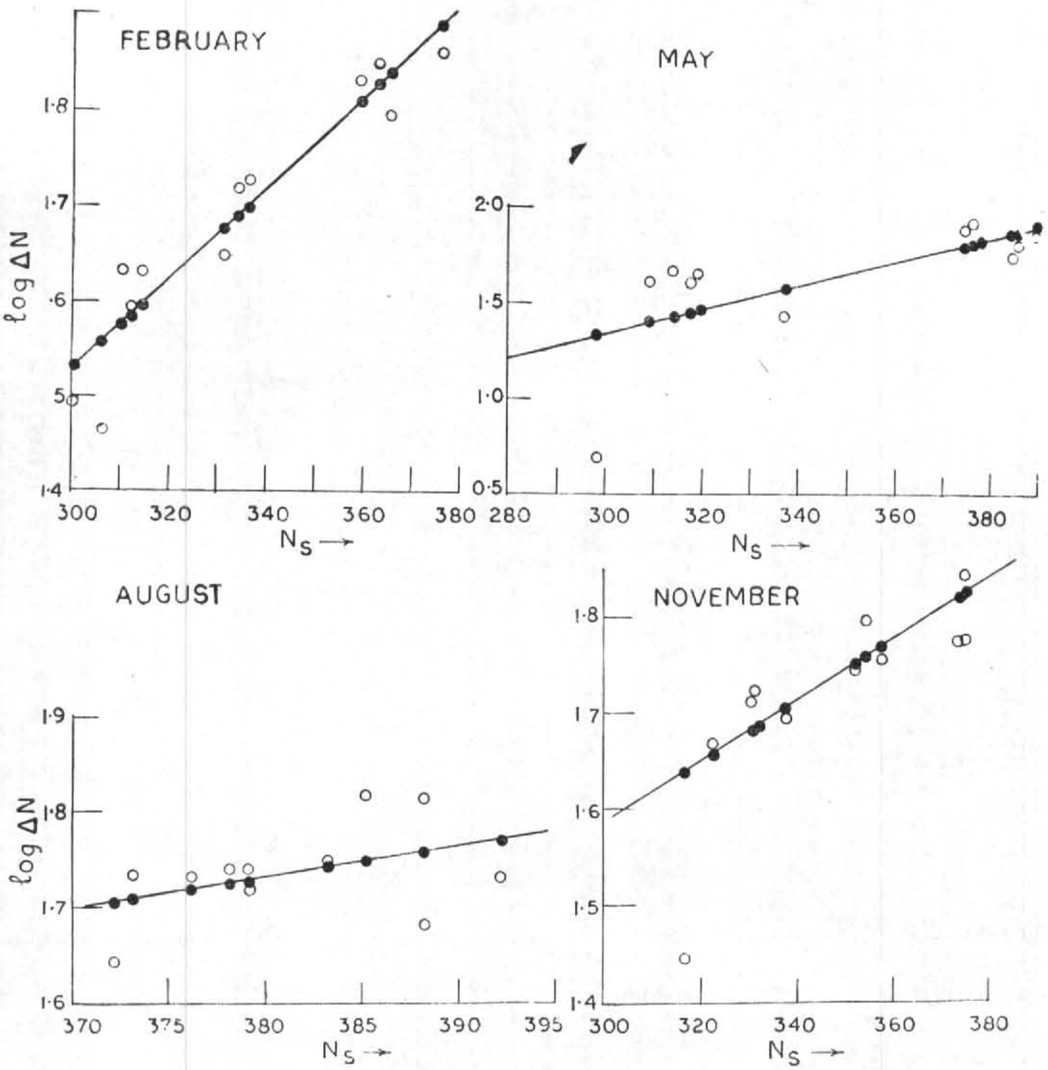

Fig. 1. Plot of log $\triangle N$ *vs* N_s

The intercept, $\log_{10} A$ in the month of May was negative, and hence the relationship between $\log \triangle N$ and N_s has been expressed in linear form.

4.3. *Straight line model* — Another model of atmospheric refractivity is based on the effective earth's concept in the first kilometre. In this atmosphere the relationship between N_s and $\triangle N$ may be expressed as —

$$\triangle N = C + D\, N_s \tag{6}$$

on the basis of a high degree of correlation between $\triangle N$ and N_s over India (Table 2). Here C and D are constants which can be determined by the least squares method.

The following equations were obtained —

$$\left.\begin{array}{ll} \triangle N = 0{\cdot}5406\, N_s - 130{\cdot}06 & \text{February} \\ \triangle N = 0{\cdot}5075\, N_s - 125{\cdot}91 & \text{May} \\ \triangle N = 0{\cdot}4093\, N_s - 101{\cdot}19 & \text{August} \\ \triangle N = 0{\cdot}4148\, N_s - 89{\cdot}98 & \text{November} \end{array}\right\} \tag{7}$$

From Eq. (7) it is seen that for a given N_s, $\triangle N$ is found to be highest during February and least during August. Similarly, for a given value of $\triangle N$, N_s is seen to be largest during August and least during February. On the basis of Eq. (5), almost similar results are obtained except that for a given $\triangle N$, N_s is lowest during May. The relationship between $\log \triangle N$ *vs* N_s and $\triangle N$ *vs* N_s is shown in Figs. 1 and 2 respectively.

4.4. *Chi-square* (χ^2) *test of significance* — The values of $\triangle N$, for a given N_s obtained with the help of Eqns. (5) and (7) are given in Table 1.

The value of χ^2 is given by

$$\chi^2 = \sum_{i=1}^{n} \frac{(\triangle N_i - \triangle N)^2}{\triangle N_i} \tag{8}$$

where n is the degree of freedom and $\triangle N_i$ is the expected value of $\triangle N$ calculated from Eq. (5) or (7).

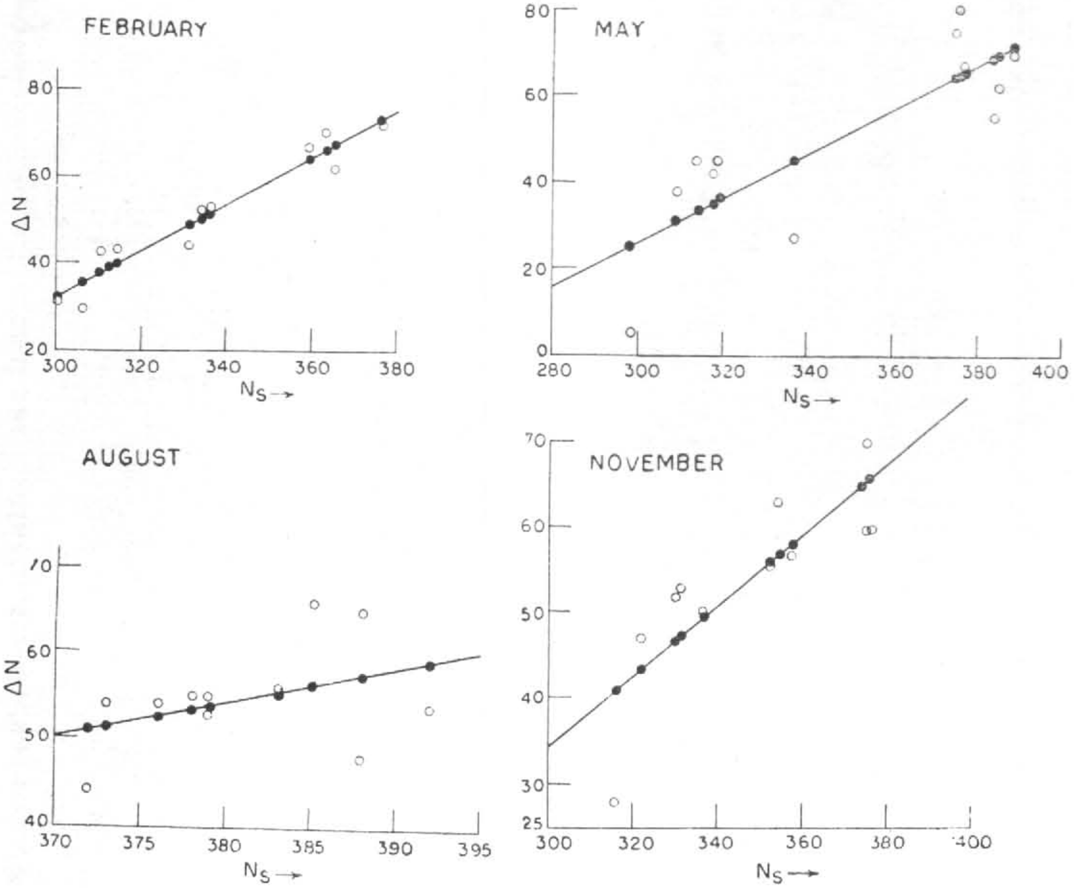

Fig. 2. Plot of $\triangle N$ vs N_s

χ^2 values calculated with the help of Eq. (8) individually for exponential and straight line expressions are given in Table 2. It may be seen that very high degree of probability is obtained showing the suitability of either type of model atmospheres in all the months except May when the scatter of data points was quite large. Although exponential and straight line fit have shown a high degree of probability during August, the correlation coefficient of the order of 0·40 forbids the use of both these types of model in this season.

5. Conclusions

The above study has shown that exponential or effective earth's concept of radio atmosphere can be used in India in the lowest one kilometre in all the representative months except August when a new type of model has yet to be investigated. However, because of the marked influence of tropical maritime air over the coastal stations in the Peninsula in the remaining months, model radio atmospheres may be derived on the basis of homoclimes of radio climate of $\triangle N$ rather than a single model over the whole of the country.

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