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ON THE THEORY OF THE GENERAL CIRCULATION

1. In 1947 C. G. Rossby published his well known theory of the general circulation of the atmosphere in terms of the conservative property of the absolute vertical vorticity. Several objections have been raised to this theory (Queney 1959) which we think merit some detailed discussion. We may summon-up the arguments opposed in the following three items :

(1) Rossby does not consider the validity of the geostrophic approximation and he is unable to obtain the Reynolds force responsible for the maintenance of the zonal winds.

(2) If Rossby's theory is right, polar east winds must exist due to the conservation of the angular momentum of the atmosphere, which are unobserved.

(3) The uniformity of ζ (absolute vorticity) is impossible because of the interaction between different scales of turbulence.

In what follows we will examine these items and try to show their inanity.

2. *The geostrophic approximation and Reynolds forces* — On account of the similarity existing between the vorticity transfer and the transfer of momentum, we may write (Belinskii 1961) :

$$-\overline{v'\,\zeta'} = -\frac{d}{dy}\,\overline{(u'\,v')} = -\frac{1}{\rho}\,\frac{d}{dy}\,\tau_{xy} \qquad (1)$$

y is directed towards the north and u', v' are the components of the eddy velocities. τ_{xy} = Reynolds tension (ρ is supposed to be independent of y).

If ζ is constant we have $\overline{v'\,\zeta'} = 0$ since $\overline{\zeta'} = 0$ and accordingly,

$$\tau_{xy} = \text{Constant} \qquad (2)$$

The total Reynolds force taking into account the sphericity of the earth, is given (Queney 1960) by :

$$F = \frac{1}{\rho}\left(\frac{\partial}{\partial y} - \frac{2\,t_g\,\phi}{\gamma}\right)\tau_{xy} \qquad (3)$$

where ϕ = latitude and γ = distance to the centre of the earth.

Reynolds forces are not essentially different from other viscous forces, what happens is that if Eq. (2) is introduced into Eq. (3) in the case τ_{xy} is positive; F is rendered negative and equal to :

$$F_b = -\frac{2\,t_g\,\phi}{\rho\,\gamma}\,\tau_{xy} \qquad (4)$$

Eq. (4) can be properly called the Rossby force and it is responsible for the maintenance of the zonal winds. The geostrophic approximation follows immediately since friction is non-existent.

3. *Conservation of the angular momentum of the atmosphere* — The total angular momentum of the atmosphere is not necessarily conserved, the earth not being spherically symmetric. Macroturbulence is not isotropic, that is invariant to rotations around any given axis, due to the action of the gravity field, so in general there will be no conservation of angular momentum. Objection No. 2 is then unapplicable.

4. *Dissipation of* ζ — This is a minor point at issue, the parallelism of the wind to the isobars being one of the prominent features of the general circulation, friction forces are negligible. The latter depend not only on the magnitude of the viscous coefficients but also of the velocity field arrangement. Transfer of energy tends to be conservative and the circulation theorem can be applied provided we limit ourselves to a barotropic atmosphere.

L. COVEZ

Department of Mathematics,
University of Paris VI, France
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