Studying monthly rainfall over Dibrugarh, Assam: Use of SARIMA approach

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ABSTRACT. Perceptible the rainfall pattern is tough for the solution of several regional environmental issues of water resources management, with implications for agriculture, climate change, and natural calamity such as floods and droughts. Statistical computing, modeling and forecasting data are key instruments for studying these patterns. The study of time series analysis and forecasting has become a major tool in different applications in hydrology and environmental fields. Among the most effective approaches for analyzing time series data is the ARIMA (Autoregressive Integrated Moving Average) model introduced by Box and Jenkins. In this study, an attempt has been made to use Box-Jenkins methodology to build ARIMA model for monthly rainfall data taken from Dibrugarh for the period of 1980-2014 with a total of 420 points. We investigated and found that ARIMA (0, 0, 0) (0, 1, 1)12 model is suitable for the given data set.

Key words – Seasonal Autoregressive Integrated Moving Average (SARIMA), Additive decomposition, Q-Q plot, Normwin.test.

1. Introduction

The hydrological cycle, regulated mainly by rainfall, is the most important subsystem of the Earth’s climate system for life on the planet. Rainfall affects the environment and society in various ways ranging from water availability for livelihood and agriculture to the functioning of various industries, hydroelectric power generation etc and thus affect the economy of a country like India. Rainfall is an important parameter affected by global climate change. The global climate changes may influence long-term rainfall patterns impacting the availability of fresh water along with the danger of increasing occurrences of droughts and floods. For example, Gu et al. (2007) have reported increasing levels of rainfall in the tropics associated with climate change (specially the global warming).

The annual migration of the Intertropical Convergence Zone (ITCZ) is the key component affecting the rainfall and thus climate in the Indian Ocean and the surrounding areas. The ITCZ migrates northward across the Indian Ocean in March-May and reaches its northernmost position during boreal June-September. During June to September, a strong low-level monsoonal air flow is generated by a strong pressure gradient between...
the low-pressure cell over the Tibetan Plateau and a high-pressure cell over the Southern Indian Ocean. North of the equator, flow of a strong southwesterly air, Somali or Findlater Jet [Findlater (1969)] transports large amount of moisture and is released as monsoon precipitation over some parts of Southern Arabia and the Indian subcontinent. The release of latent heat through condensation of moisture is an additional and important forcing of the Indian Summer Monsoon as it further strengthens and maintains the surface low pressure over the Asian landmass [Webster et al. (1998)]. During October-November, the ITCZ retreats southward and reaches its southernmost position at approximately 25° S in January. The reversed pressure gradient during the winter months generates the moderate and dry north-east monsoon [Fleitmann et al. (2007)].

The North-Eastern part of India with 66% forest cover is one of the hotspot region of rainfall in the globe, which makes it an important site from a meteorological perspective. Precipitation over this region increases sharply at the monsoon onset in May, and lasts until September. The increased rainfall over this region due mainly to orographic uplift of moisture laden air from the Bay of Bengal. The rainfall in the North-East India exhibit a peculiar pattern due to its inhomogeneous terrain and topography. As such the monsoon rainfall over this region possesses a weak out of phase relationship with that over homogeneous region of the central and western India on inter-annual time scales [Goswami et al. (2010)] and references therein). They have also reported increase in extreme events of heavy rainfall over the region during last three decades. Knowar et al. (2012), have found an East-West asymmetry (centred at 85° E) of Indian Summer Monsoon Rainfall over south Asia, western region showing increasing trend in monsoon rainfall while eastern region including the north-East India has shown a decreasing trend. Such a decreasing rainfall will result in decreasing water availability thereby affecting mainly the agriculture crucially. Again North-Eastern state Assam is prone to heavy flood during monsoon season of every year, which affects the life and property of population as well as the economy of the state drastically. As monsoon rain significantly determines the intensity and amplitude of flood in this region, so it is probable that reasonably appropriate rainfall prediction can help out the society and the state from heavy lost of wealth and economy every year.

Several investigators have used the Seasonal Autoregressive Integrated Moving Average (SARIMA) model for statistical analysis and prediction of various species. For example Dan et al. (2014); Shumway and Stoffer (2006); Ediger and Akbar (2007); Kaushik and Singh (2008); Momani (2009); Abdul-Aziz et al. (2013); MuttabeAlhashimi (2014) etc. have used this technique. Roy and Das (2012) used the SARIMA model for statistical analysis of temperature in Dibrugarh district. However, there is a lapse of time series analysis and forecasting of meteorological data in Dibrugarh district. Keeping this point in mind, in this paper, we have examined the monthly rainfall (in mm) pattern of Dibrugarh, a location in the North-East India (section 2) for the period 1980 - 2014. An attempt to forecast the rainfall for successive year has also been made using the SARIMA statistical model (section 3).

2. The study location and prevailing meteorology

The present study location (Dibrugarh; 27.4° N, 94.9° E, 111 m amsl), situated on the southern bank of river Brahmaputra in eastern Assam, close to the North-Eastern boundary of the Indian subcontinent. The Dibrugarh district extends from 27° 5’ 38” N to 27° 42’ 30” N latitude and 94° 33’ 46” E to 95° 29’ 8” E longitude. It is bounded by Dhemaji district on the North, Tinsukia district on the East, Tirap district of Arunachal district on the South-East and Sibsagar district on the North and South-West. The area stretches from the North Bank of the mighty Brahmaputra, which flows a length of 95 km through the northern margin of the district, to the Patkai foothills on the South. The Burhi Dihing, a major tributary of the Brahmaputra with its network of tributaries and wetlands flows through the district from east to west.

The Himalayan foothills in the north, which are nearly 100 km away from the site and the other hills and mountain ranges in the east and south prevent the rain bearing monsoon winds from escaping this region on one hand, while they do not allow the dry and cold winds of central Asia to enter the northeast region. The Dibrugarh town experiences mild climate with low temperature and high rainfall. On the basis of the climatic characteristics such as distribution of temperature, rainfall, rainy days, humidity, presence of fogs and thunderstorms, the climate of the area may be classified into four seasons: (a) Winter (b) Pre-monsoon (c) Monsoon and (d) Retreating monsoon.

(a) Winter (December-February) : The winter covers the months of December, January and February. In this season, fair weather prevails occasionally associated with fogs and haze. December and January are the driest months and January is the coldest. The minimum temperature ranges between 8 °C and 10 °C and the maximum between 27 °C and 29 °C. The average rainfall in the season is 20 cm.

(b) Pre-Monsoon (March-May) : The months of March, April and May constitute the pre-monsoon season. From
March the land surface gets steadily heated and the temperature starts rising. Strong convection develops due to the local depressions formed especially in the afternoon. The nor'westers locally called Bordoichilla appears during the period. Rainfall ranges between 59 and 160 cm and maximum temperature ranges between 28 °C and 32 °C. This season is, in fact, a transitional phase between the dry cool winter and the warm moist monsoon.

(c) Monsoon (June-September): With the onset of monsoon in early June, heavy rainfall occurs. Widespread low clouds and high humidity together maintain almost uniform temperature over the area. The maximum temperature ranges between 33 °C and 37 °C. The average annual rainfall during the period is 300 cm. The occurrence of thunderstorms is the most conspicuous characteristics of the monsoon weather. This is the season of dominant agricultural operation in the area.

(d) Retreating Monsoon (October-November): The monsoon withdraws from the area in the last week of September or first week of October. The cool northeasterly winds originating over the lofty mountains of the Arunachal Himalayas brings the temperature down. The orographic low is replaced by high pressure and a flat pressure gradient occurs. Rainfall decreases abruptly and the sky becomes progressively clear. Sunny days prevail till the end of November.

3. Data and methodology

The data used in this paper are completely secondary in nature and collected from two weather stations, one is Mohanbari under India Meteorological Department and another is Dibrugarh University under Indian Space Research Organisation. The two stations are within a circle of ~5 km radius from the Dibrugarh town. We, therefore, can assume that the two stations measure same precipitation and hence we can pool the data of two stations. In this study, approximately 3% of observations are found to be missing. When a series does not have too many missing observations, it may be possible to perform some missing data analysis, estimation, and replacement. A crude missing data interpolation method is used in this study. We have taken the mean for the overall series in the place of missing data.

The methodology and the theorems propounded by Box and Jenkins (1970) called the Seasonal Autoregressive Integrated Moving Average (SARIMA) has been used. This is an advance technique of forecasting requires long seasonal time series data. This model decomposes historical data into an Autoregressive (AR) process, where there is a memory of past values, an Integrated (I) process, which accounts for stabilizing or making the data stationary plus a Moving-Average (MA) process, which accounts for previous error terms making it easier to forecast. The steps involving in Box and Jenkins (1970) methodology are given below:

**Phase 1**

(a) Data Preparation: Transform data to stabilize variance and difference data to obtain stationary series.

(b) Model Selection: Examine autocorrelation function (ACF) and partial autocorrelation function (PACF) to identify potential models.

**Phase 2**

(a) Estimation: Estimate parameters in potential models. Select the best model using suitable criterion.

(b) Diagnostic: Check ACF and PACF of residuals. Examine residuals follow white noise or not. If it does not follow white noise, select another model by model selection criterion.

**Phase 3**

(a) If residuals follow white noise, use the model for forecasting.

3.1 Multiplicative seasonal autoregressive integrated moving average

The multiplicative seasonal autoregressive integrated moving average (SARIMA) model, of Box and Jenkins (1970) is given by

$$\Phi_p(B^s)\phi(B)\nabla^d\nabla^s X_t = \mu + \Theta_q(B^s)\theta(B)e_t$$

where $e_t$ is the usual white noise process. The general model is denoted by ARIMA (p, d, q) (P, D, Q)$_S$. The ordinary autoregressive and moving average components are represented by the following polynomials $\phi(B)$ and $\theta(B)$ of orders p and q, respectively,

$$\phi(B) = 1 + \phi_1B + \phi_2B^2 + \ldots + \phi_pB^p$$

$$\theta(B) = 1 + \theta_1B + \theta_2B^2 + \ldots + \theta_qB^q$$

and the seasonal autoregressive and moving average components are represented by the following polynomials $\Phi_p(B^s)$ and $\Theta_q(B^s)$ of order P and Q respectively,
\begin{align*}
\Phi_s(B^s) & = 1 - \Phi_s B^s - \Phi_{s2} B^{2s} - \ldots - \Phi_{sL} B^{Ls} \\
\Theta_s(B^s) & = 1 + \Theta_{s1} B^s + \Theta_{s2} B^{2s} + \ldots + \Theta_{sL} B^{Ls}
\end{align*}

(4)

(5)

Seasonal difference components are represented by:

\[ \nabla^d = (1 - B)^d \] and \[ \nabla^q = (1 - B^q)^d \]

3.2 Estimation of multiplicative seasonal ARIMA\((p, d, q)(P, D, Q)\)

There are several methods such as method of moments, maximum likelihood, and least squares that can be employed to estimate the parameters in the tentatively identified model. Box and Jenkins (1970) favour estimates chosen according to the maximum likelihood (ML) criterion. In ML method, we maximize likelihood function or equivalently log likelihood function with respect to parameters to be estimated. Statisticians frequently prefer the ML approach in estimation problems because the resulting estimates often have attractive statistical properties like unbiasedness and efficiency. However, finding ML estimates of SARIMA model is cumbersome due to presence of non linear equations and so it, requires software. In this study, we have tried to estimate the parameters by ML method using R software.

Let \( w_t \) be the transformed series of \( \nabla^d \nabla^q x_t \) form an ARIMA\((p,q) \times (P,Q)\) process and \( \{e_t\} \) has 0 mean and variance \( \sigma^2 \). The joint density is given by Box and Jenkins (1970):

\[
f_{\tilde{w}}(w; \phi, \theta, \Phi, \Theta, \sigma) = (2\pi \sigma^2)^{-n/2} |M_n|^{-1/2} \exp(-S/2\sigma^2)
\]

where,

\[ S = S(\phi, \theta, \Phi, \Theta, \sigma) = \sum_{t=\phi}^{\theta} \left[ E[e_t w; \phi, \theta, \Phi, \Theta, \sigma] \right] \]

is the unconditional expectation of \( e_t \) given \( \phi, \theta, \Phi, \Theta, \sigma \) and \( w \). Also \( \sigma^2 M^{-1} = \sigma^2 M^{-1}(\phi, \theta, \Phi, \Theta) \) is the covariance matrix of the difference series.

The log likelihood is given by :

\[
L = \text{Const} - (1/2) \ln(2\pi \sigma^2) - \frac{S}{2\sigma^2}
\]

(7)

Following Box and Jenkins (1970), we will assume that for moderate and large values of \( n \), the term \( (1/2) \ln \left| M_n \right| \) in Eq. (7) is dominated by \( (S/2\sigma^2) \). Discarding this term, we approximate Eq. (7), by

\[
\hat{\sigma}^2 = \frac{S}{n}
\]

(9)

4. Results and discussion

The long term rainfall records consisting of monthly total rainfall data for 35 years, starting on January 1980 to December, 2014 has been used in this study. We have plotted year in \( X \)-axis and observed rainfall data in \( Y \)-axis in Fig. 1. But it seems difficult to determine the trend of the data set whether it is upward, downward or no trend. Therefore, to examine the trend, we decompose the data by additive decomposition method by the statistical software R as depicted in Fig. 1. Thus, a
slight downward trend of rainfall pattern from 1980 to 2014 could be obtained. From Fig. 1, it is also observed that the presence of strong seasonal cycle in the rainfall data set.

Fig. 2 consists of plots of ACF and PACF taking 48 lag values in X-axis and autocorrelation values in Y-axis for the monthly rainfall series. Here a slowly declining sinusoidal wave followed by ACF and strong significant peaks at seasonal lag values 6, 12, 24, 48 with alternating sign have been observed. This alternating sign signifies that if negative correlation occurs at lag value 6 then next positive significant value occurs at lag 12 and so on. Note that autocorrelation at lag 0 is always 1 by definition. This indicates strong seasonality and non-stationarity of the data set. According to B-J methodology we must ensure that the time series being analyzed is stationary before we fit SARIMA model. In order to obtain a stationary series, we decide to first take 12-month differences of data to remove the seasonal influence. We plot the ACF and PACF for the differenced series by using the transformation 

\[ t t 12 X = t - 12 X_{t-12} \]  

(Fig. 3).

In Fig. 3, it is observed that the ACF has a single negative spike at the seasonal lag 12 and the PACF exponentially decrease at the seasonal level. In both ACF and PACF, no other significant autocorrelation at non-seasonal lag values are observed. So, we might tentatively conclude that the time series values are described by seasonal moving average model of order Q = 1, i.e., (0, 0, 0) (0, 1, 1)\( _{12} \).

For the ARIMA (0, 0, 0) (0, 1, 1)\( _{12} \) model obtained above, we estimate the parameters by using the theory as mentioned in section 3.2. The maximum likelihood estimate of \( \Theta \) obtained from R software is as follows:

\[ \hat{\Theta} = -0.8580 \text{ (s.e. } = 0.0336) \]

and then estimated model from Eq. (1) is given by

\[ (1 - B^{12})x_t = [1 + (-0.8580)B^{12}]e_t, \]  

which can be written as

\[ x_t = x_{t-12} + e_t + 0.8580e_{t-12} \]  

(10)

The coefficient is significant. The constant term of the model is omitted due to seasonal difference.

If the model fits well, the standardized residuals estimated from this model should behave as an i.i.d. (independent and identically distributed) sequence with mean zero and variance \( \sigma^2 \). Such a sequence is referred to as white noise. Fig. 4 displays a plot of the standardized residuals, the ACF of the residuals and the p-values of the Q-statistic at lag 1 through 12. From standardized plot of residuals, it is observed that little amount of residuals outside the limit of -3 and +3 which is accounted as outliers for the model. Also, this figure shows, none of the autocorrelations is individually statistically significant and nor the Ljung - Box - Pierce Q-statistics are statistically significant. We cannot reject the null hypothesis of independence in this residual series. Using the white noise test (from the normwn.test package in R : Perform a
univariate test for white noise), we obtain the $p$-value of 0.5711 which means the residuals series is white noise (with mean 0 and variance $\sigma^2$). In addition to this, a normal probability plot or a Q-Q plot can help in identifying departures from normality (Fig. 5). As we can see from Fig. 5, the residuals are departed from normal distribution, although we cannot reject the model.

Because, we checked normality of residuals for all possible values of $p$, $d$, $q$, $P$, $D$, $Q$ and found that residuals do not follow normal distribution. Among them, residuals
for the above mentioned model give better result. Hence, we need not look for another ARIMA model.

Now, the actual observations are plotted with predicted values from 1980-2014 in Fig. 6 where red line represents actual rainfall values whereas black point represents predicted. Similarly, we plot predicted values of complete 2015 in Fig. 7 with actual observations of 2015. From Fig. 7, it is observed that the amount of predicted rainfall of 2015 is less than the actual values in most of the cases (months). However, the predicted rainfall shows same picture as in the case of actual rainfall data.

5. Conclusion

In this paper, the monthly rainfall record in the Dibrugarh region has been studied using the Box-Jenkins (SARIMA) methodology. The estimation and diagnostic analysis results revealed that the models’ are adequately fitted to the historical data. The residual analysis, confirmed that there is no violation of assumptions in relation to model adequacy except normality of residuals is not full filled. The reason may be due to the presence of missing values in the original data set which are interpolated by software. Our selected ARIMA (0, 0, 0) (0, 1, 1)12 model give us one year predicted monthly rainfall that can help decision makers to establish strategies for Dibrugarh, Assam, India. The forecasted rainfall data revealed a decline rainfall during upcoming year 2015 from the previous years. This may be consistent with the decadal declining rainfall trend as discussed in the introduction section. It may be noted that this model predicts the pattern but sometimes there is difference between the actual and predicted values. As such the predicted rainfall pattern may add to the existing knowledge of weather forecasting in North-East India, particularly in Assam where flood is a major challenge for the society every year. Weather forecaster in this region may be benefited with the presented statistical model.

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Reference


